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SAMPLING FROM A DISCRETE UNIVERSE

I. Sampling without replacement from a finite universe

By LANCELOT HOGBEN and
KENNETH W. CROSS

1. Introduction.

The founding fathers of the theory of probability concerned themselves largely with the class of problems which we now call sampling with or without replacement from a finite universe. Since *de Moivre* and *Laplace* developed the normal curve as a limit of the binomial, theoretical statistics has been largely occupied with the consequences of sampling from a hypothetical normal universe, i.e. a universe of which the number of score classes is *ex hypothesi* infinite, and hence of a universe in which the replacement condition is ordinarily irrelevant since the extraction of a finite sample therefrom does not materially change its composition. Though no continuous distribution is consistent in the last resort with a particular view of matter, the use of such is not uncommonly satisfactory in practice as a descriptive device; but we should not lightly dismiss the obligation to explore the empirical credentials of an arbitrary postulate because it happens to be mathematically convenient to invoke it. There arise in practice many situations w.r.t. which it is not merely improper to assume a *normally* constituted universe as sufficiently descriptive of the reservoir from which we sample, but equally improper to assume that the extraction of a sample therefrom does not materially change its composition. Such situations may arise in statistical inspection (*Quality Control*) and in the selection of statistical documents (e.g. *hospital records*). It has special importance in that branch of statistical genetics designated by *Dahlberg* as the *theory of isolates*.

The neglect of the theory of sampling without replacement is the more remarkable because we may regard sampling with replacement

as the limiting case of non-replacement sampling when the universe becomes infinite. If we speak of a finite universe, we commonly imply a universe made up of a finite number (N) of items to each of which we can attach a score value; but if N is sufficiently large in comparison with the size (r) of a sample taken therefrom, no sensible error results from the assumption that the composition of the residual universe is the same as the universe before extraction. Within such an assemblage of discrete score values we may assign items which bear the same score to a particular class, the possible number (n) of score classes being equal to, as is true of the rectangular universe, or less than N. Whether a normal, or other continuous, distribution will prove to be a satisfactory approximate description of a universe so conceived depends primarily upon whether the number of score classes is large; but an indefinitely large value of N is consistent with a very small value of n.

For the binomial universe of 2 score classes $n = 2$; and we are free to conceive it in terms of an enumerable infinity of items or of a finite number $N \geq 2$. So conceived, the numbers of items (n_p and n_q) respectively allocated to one or other class may each be an enumerable infinity, though the ratios p and q defined by the relations $Np = n_p$ and $Nq = n_q$ are themselves finite. This is the assumption on which we proceed to derive the sampling distribution referable to success and failure of a treatment in a clinical trial. In contradistinction to the Finite Universe of which the distribution of score classes, henceforth called the *unit-sample distribution*, is necessarily discrete, a universe may thus be both discrete and infinite. If so, the replacement condition is trivial; but we shall expect to obtain a good descriptive curve for the distribution of the r-fold sample only if r is of sufficient size to accommodate many consecutive score classes.

The operations exactly descriptive of sampling from a universe w.r.t. which n alone is finite or of a universe w.r.t. which both n and N are finite equally belong to the domain of the *finite* calculus; but an exact solution may be impossible and will be very laborious unless r is small. It is therefore of practical importance to define in what circumstances a normal or other continuous curve can give a sufficiently precise description of the sampling process. Thus the discrete universe invites enquiry of two sorts: (a) how the absolute size of the sample affects the facies of the sampling distribution when the universe itself is both infinite and discrete; (b) how the sampling fraction determines the character of the former when the latter is discrete and

finite. As regards (a), we shall later see that the infinite binominal universe merits more attention than it has hitherto received, and universes of 3 or more classes have hitherto attracted little attention. Concerning (b), it suffices to say that the hypergeometric distribution of non-replacement sampling from a 2-class universe circumscribes most of what we know already, and is even so incomplete.

With the end in view stated above, it will be convenient to employ a distinction drawn by *Hogben* and *Waterhouse* [1949] between two types of scoring they denote respectively as *taxonomic* and *representative*. These terms are more explicit for the present purpose than the customary dichotomy between sampling of attributes and sampling of measurements or between qualitative and quantitative statistics, since the definition of an attribute may be quantitative and a quantitative statistic is not necessarily metrical. An attribute, i.e. the criterion of a class, may be qualitative or quantitative in the sense that we may distinguish individuals as: (a) yellow or green, Protestant or Catholic; (b) having less than or more than 4 million blood corpuscles per mm^3, or weighing less than 5 ½ lb. at birth. Both specifications included under (b) are quantitative in the ordinary sense of the term but only one of them is metrical. In statistical problems one classifies samples: (a) by enumeration of individuals with a common attribute which we may either define in qualitative or quantitative (enumerative or metrical) terms; (b) by some representative figure (e.g. sum, mean or median) which takes account of a numerical score attached to each individual item of the universe. It is the first that we speak of below as taxonomical scoring, the second as representative.

In the particular case of a binomial universe ($n = 2$), we may score the unit sample as 0 or 1 and the taxonomic then becomes a special case of the representative, but only in virtue of the fact that every individual member of the universe being classifiable as a member of class A or *not*-A has a unit score of 0 or 1. In the same way, of course, the difference between a rectangular and a binomial universe breaks down when $q = \frac{1}{2} = p$, if successive terms of $(q + p)^1$ define the unit sampling distribution. When $n > 2$ the distinction between taxonomic scoring and representative scoring as defined above is fundamental. In the domain of binary taxonomic scoring the *raw* scores definitive of an r-fold sample run from 0,1...r by unit steps. The origin of the distribution is not necessarily zero if the method of scoring is representative; but this will not affect

the evaluation of the mean moments. The scale of the distribution of the score-sum is not necessarily unity; but this will not affect the evaluation of the β coefficients. By the same token, the β-coefficients of the mean score will be the same as those of the score-sum in the representative domain; and the same applies to the proportionate and raw scores in the binary taxonomic domain as a limiting case. Contrariwise, the algebraic properties of the distribution of the difference between the mean scores (or proportionate scores) of a-fold and b-fold samples from the same universe are not identical with those of the corresponding score-sums (or raw-scores) for reasons more fully discussed in a later communication of this series. Accordingly, it will be necessary to consider separately the difference distribution w.r.t. the score-sum (or raw score) and the mean (or proportionate) score.

In pursuing our objective, as stated above, we shall rely mainly upon the method of curve fitting by moments developed by *Karl Pearson*; but it is important to emphasise that the adequacy of a curve fitted to a discrete distribution by the Pearsonian method is an empirical issue which calls for arithmetical investigation to justify its credentials in a given situation. It is also pertinent to recall at the outset that all moments of finite order referable to a discrete universe are themselves finite.

In the Pearsonian system of model sampling distribution moments higher than those of the fourth order and hence Beta coefficients of order higher than β_2, do not occur as parameters. The justification of this scarcely calls for comment when the number of score classes in the universe is such that the contour of its histogram closely approaches that of a continuous curve. In any treatment of the finite universe, however, this postulate may be inappropriate as the following example will suffice to show. We suppose that the range of empirical observations extends only over scale divisions -1, 0, $+1$ with relative frequencies 1, 4, 1. The values of β_1 and β_2 are then respectively 0 and 3 as for the unit sampling distribution for the normal universe. As is likewise true of a normal universe, the distribution of the mean score retains the same values (of β_1 and β_2) for samples of 2, 3 or more; and all β-coefficients of odd order (β_3, β_5) are zero for samples of any size. Needless to say, however, the values of higher *Pearson*[1] coefficients of even order (β_4, β_6, etc.) for the

[1] We extend, in the manner of *Kendall* [1947], the notion of these parameters of a distribution beyond *Pearson*'s original conception.

unit sampling distribution of such a 3-class universe depart widely from their normal values. When interpreting the consequences of sampling from a finite universe, it is therefore profitable to give special attention to moments of higher order than the second. We here employ *Kendall*'s definitions, *viz.* if m_k is the k^{th} mean moment of a distribution

$$\beta_{2s-1} = \frac{m_3 \cdot m_{2s+1}}{m_2^{s+2}} \tag{1.01}$$

$$\beta_{2s} = \frac{m_{2s+2}}{m_2^{s+1}} \tag{1.02}$$

When we speak of the fitting curve for a discrete sampling distribution as satisfactory, our criterion of goodness of fit will be that it assigns to some specified range of score values a frequency which differs from its true value by an error numerically less than a predetermined figure. For instance, we may require that the frequency assigned to a range of scores from, say $-\infty$ to x with actual frequency 0.95 will lie between 0.945 and 0.955, the proportionate error being of the order of 0.5%. A paramount consideration dictating the particular distribution postulated with more or less plausibility as an approximate description of our universe is then the possibility of deducing a distribution of extracted samples in a form suitable for tabulation. When our concern, as in this communication, is with sampling without replacement our universe is finite and we may be able to specify its structure (i.e. its unit-sample distribution) exactly. A first approach to the problem of fitting to a discrete distribution a curve which is satisfactory in the sense defined above then involves a specification of the moments of the r-fold (or other) sample distribution in terms of the moments of the u-s-d. Such expressions developed below refer to scoring in the representative domain, and include the result of sampling from a finite binomial universe as a special case.

When we speak of the number (n) of score classes of the u-s-d in this context, we imply that every such class contains at least one item. If the increment of score is fixed, the number of score classes with at least one item in the replacement sampling distribution is then $r(n-1) + 1$. The number of score classes of an r-fold non-replacement sample distribution when N is finite will be less than this, if the number of items in either the lowest or highest score

valued class of the u–s–d is less than r. It therefore goes without saying that the number of score classes of the finite sample from the discrete universe, i.e. universe itself consisting of a finite number of score classes, is always finite; and that no infinite series can exactly describe sampling from such a universe.

2. *Symbolism employed.*

To take stock of sampling without replacement, it is necessary to label: (a) every item in the total universe as an individual entity regardless of the possibility that the numerical value of the score we attach to it is identical with that of any other item of the same score class; (b) every item of one sample as an entity distinct from any item present in any other sample from the same universe. We can do this by using a right hand subscript (u) to label a score (x_u) as that of an item chosen at the u^{th} draw, in which case we label the scores of the residual items in the (n–a)-fold universe after extraction of an a-fold sample as $x_{a+1}, x_{a+2} \cdot \cdot \cdot \cdot \cdot x_{n-1}, x_n$. The meaning of the subscripts $(a+1)$, $(a+2)$ etc. is neither more nor less arbitrary than the meaning we attach to the third card taken in a *simultaneous* 5-fold draw. Regardless of its denomination, we then visualise each item of the sample and of the *residual* universe as one of an arbitrary linear sequence invoked for purposes of identification of the card. If we adopt this convention for the unit score, it is convenient to designate the score sum of the first a-fold sample as ${}_ax_.$, e.g. for the score sum of the 4-fold sample, we shall write ${}_4x_. = x_1+x_2+x_3+x_4$. If we now take a second (b-fold) sample after removing a items, we may label the score-sum as ${}_bx_a$, e.g. we write for the 3-fold sample extracted after previously taking a 4-fold sample ${}_3x_4 = x_5+x_6+x_7$. For the score difference between the initial (a-fold) and subsequent (b-fold) sample score sums we shall write ${}_{(a-b)}x_. = {}_ax_. - {}_bx_a$ allowing for the possibility that we may need to substract the score sums of samples each taken subsequently to another, e.g. ${}_{(b-c)}x_a = {}_bx_a - {}_cx_{b+a}$.

To express moments economically, it will be convenient to adopt a fixed convention for sums of powers and powers of sums, *viz*: (a) for the sum of the k^{th} powers of the a-fold sample ${}_ax_.^k$, so that ${}_4x_.^3 = x_1^3+x_2^3+x_3^3+x_4^3$; (b) for the kth power of the a-fold sample score sum $({}_ax_.)^k$, so that $({}_4x_.)^3 = (x_1+x_2+x_3+x_4)^3$. For the unit sample the brackets are redundant, i.e. $x_u^k = (x_u)^k$. For the k^{th} moment about zero of the total universe of scores, henceforth designated *zero moments* of the unit sample distribution, we shall

write $\mu_k = E(x_1^k)$, for the distribution of the score sum of the first a-fold sample ${}_a\mu_k = E({}_ax_.)^k$ and for that of the distribution of the score sum of a subsequent b-fold sample ${}_{b.a}\mu_k = E({}_bx_a)^k$. For *mean* moments (i.e. moments about the mean) we shall employ m_k, ${}_am_k$, ${}_{b.a}m_k$ in the same way.

The following relations summarise the foregoing definitions:

$$ {}_aX_.^k = \sum_{u=1}^{u=a} x_u^k \quad ; \quad {}_bX_a^k = \sum_{u=a+1}^{u=a+b} x_u^k \tag{2.01} $$

$$ {}_{(a+b)}X_.^k = {}_aX_.^k + {}_bX_a^k = \sum_{u=1}^{u=a+b} x_u^k \tag{2.02} $$

$$ {}_{(a-b)}X_.^k = {}_aX_.^k - {}_bX_a^k = \sum_{u=1}^{u=a} x_u^k - \sum_{u=a+1}^{u=a+b} x_u^k \tag{2.03} $$

$$ {}_{(b-c)}X_a^k = {}_bX_a^k - {}_cX_{a+b}^k = \sum_{u=a+1}^{u=a+b} x_u^k - \sum_{u=a+b+1}^{u=a+b+c} x_u^k \tag{2.04} $$

$$ \mu_k = E(x_1^k) = \frac{I}{N} \sum_{u=1}^{u=n} x_u^k = \frac{I}{N} \cdot {}_NX_.^k \tag{2.05} $$

$$ \therefore \quad {}_NX_.^k = N.\mu_k \tag{2.06} $$

$$ {}_a\mu_k = E({}_ax_.)^k \tag{2.07} $$

$$ {}_{(a+b)}\mu_k = E({}_{(a+b)}x_.)^k \tag{2.08} $$

$$ {}_{(a-b)}\mu_k = E({}_{(a-b)}x_.)^k \tag{2.09} $$

In the above the operation $E(\ldots)$ signifies, as customarily, taking the mean of all possible values of the argument. In what follows, we may sidestep unnecessary labour, when it is necessary to investigate the relevance of *choice-order* by a partial notation. $E_{v.u}(\ldots)$ signifies its mean value for a fixed value of x_u, and $E_{u.v}(\ldots)$ for a fixed value of x_v. In the same way, $E_{w.uv}(\ldots)$ signifies the mean value of a function of x_u, x_v, x_w for fixed values of both x_u and x_v.

In this notation

$$E_u.E_{v.u}(\cdots) \equiv E(\cdots) \equiv E_v.E_{u.v}(\cdots)$$

$$E_u.E_{v.u}.E_{w.uv}(\cdots) \equiv E(\cdots) \equiv E_v.E_{u.v}.E_{w.uv} \text{ etc.} \qquad (2.10)$$

The order of the appropriate operations is immaterial. In this symbolism $E_u(f) \equiv E(f)$ if f is a function of u alone.[1]

3. Moments of the r-fold sample from the infinite discrete distribution.

Though our main concern in this communication is sampling *without* replacement from a finite universe, it will clarify our task if we first indicate a method by which it is possible to specify the *Pearson* coefficients of the score sum or mean score of a distribution which is discrete but not finite or of a finite universe on the assumption that the replacement condition holds good. For any discrete universe moments of finite order are finite and the k^{th} moment of the distribution of the (a+1)-fold sample score-sum is in the notation of § 2:

$$_{a+1}\mu_k = E(_{a+1}x_.)^k = E(_ax_. + x_{a+1})^k$$

$$\therefore \quad _{a+1}\mu_k = \sum_{w=0}^{w=k} k_{(w)}\, E(_ax_.)^w\, (x_{a+1})^{k-w} \qquad (3.01)$$

If the replacement condition holds good, the value of the $(a+1)^{th}$ unit sample (x_{a+1}) does not depend on the score sum $(_ax_.)$ of the antecedent a-fold sample, and its mean value is that of the first unit sample, i.e.

$$E(_ax_.)^w(x_{a+1})^{k-w} = E(_ax_.)^w.\, E(x_{a+1})^{k-w} = E(_ax_.)^w.\, E(x_1)^{k-w}$$

$$= {}_a\mu_w \cdot \mu_{k-w}$$

Whence we derive:

$$_{a+1}\mu_k = \sum_{w=0}^{w=k} k_{(w)} \cdot {}_a\mu_w \cdot \mu_{k-w} \qquad (3.02)$$

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[1] This notation is consistent with the customary convention for partial correlation and equally appropriate to partial differentiation, e.g. $D_{x.y}(z)$ for the partial differential of $z = f(x, y)$ with respect to x and $D_{y.x}$ for the partial differential of z with respect to y; similarly, $D_{x.yz}$ for the partial differential of $u = f(x, y, z)$ with respect to x and so forth.

In 3.01 and elsewhere we follow Aitken, i.e. $k_{(w)} = k! \div w!\,(k-w)!$

When there is replacement we may write the score of the unit sample from the u-s.d. mean as $X_1 = (x_u - \mu_1)$ and that of the score sum of the a-fold sample from its mean value, which is $E({}_ax_.) = a.\mu_1$, as ${}_aX_. = ({}_ax_. - a.\mu_1)$, whence ${}_{a+1}X_. = {}_aX_. + X_1$.
For the k^{th} mean moment of the (a+1)-fold sample score-sum distribution we then have:

$$ {}_{a+1}m_k = E({}_aX_. + X_1)^k $$

$$ \therefore \quad {}_{a+1}m_k = \sum_{w=0}^{w=k} k_{(w)} \cdot {}_am_w \cdot m_{k-w} \qquad (3.03) $$

For the 2-fold sample (a = 1) the foregoing expressions (3.02) and (3.03) involve only moments of the unit sample distribution, i.e. μ_w and μ_{k-w} or m_w and m_{k-w}. By iteration we then see that we can expand the moments of the a-fold sample score sum in terms of a series involving moments of the unit sample with coefficients generated by the algorithms for summation of figurate numbers. We then find that:

$$ {}_am_2 = a.m_2 \qquad (3.04) $$

$$ {}_am_3 = a.m_3 \qquad (3.05) $$

$$ {}_am_4 = a.m_4 + 3a^{(2)}m_2^2 \qquad (3.06) $$

$$ {}_am_5 = a.m_5 + 10a^{(2)}m_3.m_2 \qquad (3.07) $$

$$ {}_am_6 = a.m_6 + 15a^{(2)}m_4.m_2 + 10a^{(2)}m_3^2 + 15a^{(3)}m_2^3 \qquad (3.08) $$

$$ {}_am_7 = a.m_7 + 21a^{(2)}m_5.m_2 + 35a^{(2)}m_4.m_3 + 105a^{(3)}m_3.m_2^2 \qquad (3.09) $$

$$ {}_am_8 = a.m_8 + 28a^{(2)}m_6.m_2 + 56a^{(2)}m_5.m_3 + 35a^{(2)}m_4^2 + 210a^{(3)}m_4.m_2^2 + 280a^{(3)}m_3^2.m_2 + 105a^{(4)}m_2^4 \qquad (3.10) $$

If we use the symbol ${}_a\beta_r$ for the *Pearson* coefficient of order r referable to the a-fold mean score or score sum distribution and β_r for the corresponding *Pearson* coefficient of the unit sample distribution, we thus derive[1]:

[1] The expressions for the moments in 3.04 to 3.10 of the discrete distribution tally with those given by *Irwin* on the more restrictive assumption of a continuous distribution of which all the moments are finite. But the derivation by this or the alternative method cited below is more general and we may regard the corresponding expressions for moments of the mean from a continuous u-s-d as a limiting case.

$$ {}_a\beta_1 = \frac{1}{a}\,\beta_1 \tag{3.11} $$

$$ {}_a\beta_2 = 3 + \frac{1}{a}\,(\beta_2 - 3) \tag{3.12} $$

$$ {}_a\beta_3 = \frac{1}{a^2}\Big[\beta_3 + 10(a-1)\,\beta_1\Big] \tag{3.13} $$

$$ {}_a\beta_4 = 15 + \frac{1}{a^2}\Big[(\beta_4 - 15) + 15(a-1)\,(\beta_2 - 3) + 10(a-1)\,\beta_1\Big] \tag{3.14} $$

$$ {}_a\beta_5 = \frac{1}{a^3}\Big[\beta_5 + 21(a-1)\,\beta_3 + 35(a-1)\,\beta_2.\,\beta_1 + 105(a-1)^{(2)}\,\beta_1\Big] \tag{3.15} $$

$$ {}_a\beta = 105 + \frac{1}{a^3}\Big[(\beta_6 - 105) + 28\,(a-1)\,(\beta_4 - 15) + 56\,(a-1)\,\beta_3 + 35\,(a-1)\,(\beta_2 - 3)^2 + 210(a-1)^2(\beta_2 - 3) + 280\,(a-1)^{(2)}\,\beta_1\Big] \tag{3.16} $$

Regardless of the character of the unit-sample distribution, the foregoing expressions for ${}_a\beta_1$, ${}_a\beta_2$ etc. all approach the corresponding β-coefficients of the normal distribution, *viz*: 0, 3, 0, 15, 0, 105, when the size (a) of the sample is large. Thus the normal curve is likely to give a good fit to the mean-score or score-sum distribution of large samples extracted with replacement from any finite universe or extracted without replacement from any discrete universe if also infinite in the sense defined in § 1. We shall later explore how large a must be when n is small.

Expressions similar to (3.04)–(3.10) are obtainable for the zero moments but will contain 2 more terms in virtue of the fact that $m_1 = 0$ for any distribution. The iterative method employed to derive (3.04) – (3.16) is more economical for the derivation of higher moments of a score-sum or mean score replacement distribution or—what amounts to the same thing—the distribution of the s-s or m-s from a discrete universe which is also infinite. For the derivation of lower moments it is not less laborious than an alternative method which is applicable to the more general case of nonreplacement sampling. In this context, it will suffice to outline the alternative procedure within the foregoing framework of assumptions, *viz.* that order of choice of the constituent unit-samples of the a-fold sample is immaterial. Whether this is so or otherwise, in our notation

$$ {}_a\mu_k = E(x_1 + x_2 + x_3 \ldots\ldots x_{a-1} + x_a)^k \qquad (3.17) $$

Expansion of the multinomial on the right leads to a series in which there will be a certain number (C_k) of terms of the form x_u^k, a certain number of terms $C_{k-1.1}$ of the form $x_u^{k-1}.x_v$ etc. and we may express this as follows:

$$ (x_1 + x_2 \ldots\ldots x_a)^k = C_k.x_u^k + C_{k-1.1}.x_u^{k-1}.x_v + C_{k-2.2}.x_u^{k-2}.x_v^2 $$
$$ + C_{k-2.1.1}.x_u^{k-2}.x_v.x_w \text{ etc.} $$

If the replacement condition holds good—as it must when the universe is infinite

$$ E(x_u^b.x_v^c.x_w^d \ldots) = E(x_u^b).E(v_v^c).E(_w^d) \ldots = \mu_b.\mu_c.\mu_d \qquad (3.18) $$

Hence (3.17) becomes:

$$ {}_a\mu_k = C_k.\mu_k + C_{k-1.1}.\mu_{k-1}.\mu_1 + C_{k-2.2}.\mu_{k-2}.\mu_2 $$
$$ + C_{k-2.1.1}.\mu_{k-2}.\mu_1^2 \ldots\ldots \text{ etc.} \qquad (3.19) $$

In this expression the coefficients C_k, $C_{k-1.1}$ etc. are determinable from elementary considerations w.r.t. number-partitions; and in this context it will suffice to cite the values of $C_{a.b.c.d}$ relevant to the multinomials of order not higher than 4 since our concern is with the evaluation of ${}_a\beta_1$ and ${}_a\beta_2$, *viz*:

$$ \left.\begin{array}{llll} C_{k.0} = a & & C_{3.1} = 4a^{(2)} & \\ C_{1.1} = a^{(2)} & & C_{2.2} = 3a^{(2)} & \\ C_{2.1} = 3a^{(2)} & & C_{2.1.1} = 6a^{(3)} & \\ C_{1.1.1} = a^{(3)} & & C_{1.1.1.1} = a^{(4)} & \end{array}\right\} \qquad (3.20) $$

By substitution of (3.20) in (3.19) we can obtain by recourse to the usual formulae for m_k in terms of zero moments the results cited by (3.04) – (3.10) above, e.g.

$$ {}_a\mu_2 = a\mu_2 + a^{(2)}\mu_1^2 \qquad (3.21) $$

$$ {}_a\mu_3 = a\mu_3 + 3a^{(2)}\mu_2.\mu_1 + a^{(3)}\mu_1^3 \qquad (3.22) $$

$$ {}_a\mu_4 = a\mu_k + 4a^{(2)}\mu_3.\mu_1 + 3a^{(2)}\mu_2^2 + 6a^{(3)}\mu_2.\mu_1^2 + a^{(4)}\mu_1^4 \qquad (3.23) $$

The foregoing derivation depends on the assumption implicit in 3.18, *viz.* that the extraction of any constituent unit sample of

the a-fold sample does not affect the expected value of any subsequent one. Before we can adapt the method to the more general case of sampling without replacement, it will be necessary (§ 4 *infra*) to remove this restriction, i.e. to find a meaning for $E(x_u^a . x_v^b . x_w^c \ldots)$ when the universe is finite and no replacement occurs. First, it is appropriate to recall that our concern in this context has been with the score-sum. Hence the signs of $x_1, x_2 \ldots x_a$ in (3.17) are all positive. When we come to consider the distribution of the score difference of 2 samples from the same universe, we shall also need to remove this restriction.

If the multinominal $(x_1 + x_2 \ldots . x_{a+b})^k$ contains $(a + b)$ terms all of which are positive, the expressions for $C_{k.0}$, $C_{k-1.1}$ etc. are obtainable for $k = 2, 3, 4$ from (3.20) by substituting $(a+b)$ for a. If a are positive and b are negative, we may write the corresponding coefficients as:

$$\left.\begin{array}{llll} H_{2.0} = (a+b) & & H_{4.0} = (a+b) & \\ H_{1.1} = (a-b)^{[2]} & & H_{3.1} = 4(a-b)^{[2]} & \\ H_{3.0} = (a-b) & & H_{2.2} = 3(a+b)^{(2)} & \\ H_{2.1} = 3a^{(2)}-3b^{(2)} & & H_{2.1.1} = 6(a^{(3)}-ba^{(2)}-ab^{(2)} + b^{(3)}) & \\ H_{1.1.1} = (a-b)^{[3]} & & H_{1.1.1.1} = (a-b)^{[4]} & \end{array}\right\} \quad (3.24)$$

In the above we have employed an economical symbolism for factorial power series involving alternate positive and negative terms. The *Vandemonde* expansion of $(a+b)^{(r)}$ in its customary form presupposes that a and b are both positive integers; but we can interpret it correctly if we write:

$$(a-b)^{(r)} = \sum_{k=0}^{k=r} r_{(k)}\, a^{(k)}\, (-b)^{(r-k)}$$

This would be strictly analogous to the binomial expansion $(a-b)^r$ if it were true that $(-b)^{(k)} = (-1)_k\, b^{(k)}$. For brevity, we shall use square brackets as above for a summation in factorial powers analogous to the expansion of $(a-b)^r$ on the assumption that a and b are both integers, *viz.*:

$$(a-b)^{[r]} = \sum_{k=0}^{k=r} (-1)^{(k)}\, r_{(k)}\, a^{(k)}\, b^{(r-k)} \quad (3.25)$$

4. Score-sum and mean score non-replacement distribution.

To employ the method of (3.17) *et seq.* to the analysis of sampling from the finite N-fold universe without replacement we have to find a meaning for $E(x_u^b . x_v^c . x_w^d \dots)$ with due regard to the fact that the value of x_u in a particular sample places a restriction on the possible value of x_v in the same one. Let us first consider the expression $E(x_u^k)$, which we may consider in this context as a function of u alone, so that $E(x_u^k) = E_u(x_u^k)$. This is the mean value of the k^{th} power of the unit sample at the u^{th} draw, i.e. after extraction of (u–1) unit samples, and hence from a residual universe containing (N−u+1) items, whence

$$E(x_u^k) = E_u(x_u^k) = \frac{E_u({}_{(N-u+1)}x_.^k)}{(N-u+1)} \tag{4.01}$$

If v = (u+1), we must interpret the operation $E_{v.u}$ to signify the mean value of the unit score from an (N–u)-fold residual universe, i.e.

$$E_{v.u}(x_v^k) = \frac{{}_{(N-u+1)}x_.^k - x_u^k}{(N-u)}$$

In the notation of § 2:

$$E(x_v^k) = E_u . E_{v.u}(x_v^k)$$

Hence from (4.01):

$$E(x_v^k) = \frac{1}{N-u}\left(1 - \frac{1}{N-u+1}\right) E_u({}_{(N-u+1)}x_.^k)$$

$$= \frac{E_u({}_{(N-u+1)}x_.^k)}{N-u+1}$$

$$\therefore \quad E(x_v^k) = E(x_u^k) = E(x_{u+1}^k)$$

$$\therefore \quad E(x_u^k) = E(x_1^k) = \mu_k \tag{4.02}$$

Thus the mean value of the k^{th} power of the unit sample extracted at any draw is the k^{th} zero moment of the unit sample distribution. To interpret product terms involving powers of different unit samples, we proceed in the same way; but shall place no restriction on either v or u other than that: (a) they are unequal; (b) each lies in the range 1 to r when r is the sample size. For a product of order (a, b) we may write

$$E(x_u^a . x_v^b) = E_u \left[x_u^a . E_{v.u}(x_v^b) \right]$$

Here $E_{v.u}(\ldots)$ signifies extracting the mean value of the score power from a universe which does not contain x_u, being therefore an (N–1)-fold universe. Since choice order does not affect its value, we may therefore write in virtue of (2.05) – (2.06):

$$E_{v.u}(x_v^b) = \frac{{}_N x^b . - x_u^b}{N-1} = \frac{N}{N-1} \mu_b - \frac{x_u^b}{N-1}$$

$$\therefore \quad E(x_u^a . x_v^b) = \frac{N}{N-1} \mu_b \, E_u(x_u^a) - \frac{1}{N-1} E_u(x_u^{a+b})$$

$$= \frac{N}{N^{(2)}} \mu_b \, E_u(x_u^a) - \frac{N}{N^{(2)}} E_u(x_u^{a+b})$$

Whence from 4.02

$$E(x_u^a . x_v^b) = \mu_{a.b} = \frac{N}{N^{(2)}} \mu_a . \mu_b - \frac{N}{N^{(2)}} \mu_{a+b} \tag{4.03}$$

In the same way, we may write:

$$E(x_u^a . x_v^b . x_c^w) = E_u \left\{ x_u^a \, E_{v.u}(x_v^b) \left[E_{w.uv}(x_w^c) \right] \right\}$$

$$E_{w.uv}(x_w^c) = \frac{{}_N x^c . - x_u^c - c_v^c}{N-2} \quad \text{etc.}$$

Similarly, we write as follows to evaluate $E(x_u^a . x_v^b . x_w^c . x_z^d)$:

$$E_{z.uvw}(x_z^d) = \frac{{}_N x^d . - x_u^d - x_v^d - x_w^d}{N-3}$$

Whence we obtain:

$$E(x_u^a . x_v^b . x_w^c) = \mu_{a.b.c} = \frac{N^3}{N^{(3)}} \mu_a . \mu_b . \mu_c$$

$$- \frac{N^2}{N^{(3)}} \left\{ \mu_a . \mu_{b+c} + \mu_b . \mu_{a+c} + \mu_c . \mu_{a+b} \right\}$$

$$+ \frac{2N}{N^{(3)}} \mu_{a+b+c} \tag{4.04}$$

$$E(x_u^a . x_v^b . x_w^c . x_z^d) = \mu_{a.b.c.d} = \frac{N^4}{N^{(4)}} \mu_a\mu_b\mu_c\mu_d -$$

$$\frac{N^3}{N^{(4)}} \Big\{ \mu_d\mu_c\mu_{a+b} + \mu_a\mu_d\mu_{b+c} + \mu_b\mu_d\mu_{a+c} + \mu_a\mu_b\mu_{c+d} + \mu_a\mu_c\mu_{b+d} + \mu_b\mu_c\mu_{a+d} \Big\} + \frac{N^2}{N^{(4)}} \Big\{ \mu_{c+d}\mu_{a+b} + \mu_{b+d}\mu_{a+c} + \mu_{a+d}\mu_{b+c} + 2\mu_a\mu_{b+c+d} + 2\mu_b\mu_{a+c+d} + 2\mu_c\mu_{a+b+d} + 2\mu_d\mu_{a+b+c} \Big\} - \frac{6N}{N^{(4)}} \mu_{a+b+c+d} \qquad (4.05)$$

From (3.17) and (3.20) we may express the first 4 zero moments of the r-fold sample in the form:

$$\left.\begin{aligned} {}_r\mu_1 &= r.\mu_1 \\ {}_r\mu_2 &= r\mu_2 + r^{(2)}\mu_{1.1} \\ {}_r\mu_3 &= r\mu_3 + 3r^{(2)}\mu_{2.1} + r^{(3)}\mu_{1.1.1} \\ {}_r\mu_4 &= r\mu_4 + 4r^{(2)}\mu_{3.1} + 3r^{(2)}\mu_{2.2} + 6r^{(3)}\mu_{2.1.1} + r^{(3)}\mu_{1.1.1} \end{aligned}\right\} \qquad (4.06)$$

From (4.03 – (4.05) we have:

$$\mu_{1.1} = \frac{1}{N^{(2)}} \Big[N^2\mu_1^2 - N\mu_2 \Big]$$

$$\mu_{2.1} = \frac{1}{N^{(2)}} \Big[N^2\mu_1\mu_2 - N\mu_3 \Big]$$

$$\mu_{2.2} = \frac{1}{N^{(2)}} \Big[N^2\mu_2^2 - N\mu_4 \Big]$$

$$\mu_{3.1} = \frac{1}{N^{(2)}} \Big[N^2\mu_1^2\mu_3 - N\mu_4 \Big]$$

$$\mu_{1.1.1} = \frac{1}{N^{(3)}} \Big[N^3\mu_1^3 - 3N^2\mu_1\mu_2 + 2N\mu_3 \Big]$$

$$\mu_{2.1.1} = \frac{1}{N^{(3)}} \Big[N^3\mu_1^2\mu_2 - 2N^2\mu_1\mu_3 - N^2\mu_2^2 + 2N\mu_4 \Big]$$

$$\mu_{1.1.1.1} = \frac{1}{N^{(4)}} \Big[N^4\mu_1^4 - 6N^3\mu_2\mu_1^2 + 3N^2\mu_2^2 + 8N^2\mu_1\mu_3 - 6N\mu_4 \Big]$$

By substitution in (4.06) we therefore obtain the first 4 zero moments of the r-fold score-sum distribution in terms of those of the unit-sample distribution, *viz.*:

$$ {}_r\mu_2 = \frac{r}{N-1}(N-r)\,\mu_2 + \frac{Nr^{(2)}}{N-1}\mu_1^2 \tag{4.08} $$

$$ {}_r\mu_3 = \frac{r(N-r)(N-2r)}{(N-1)^{(2)}}\mu_3 + \frac{3N(N-r)r^{(2)}}{(N-1)^{(2)}}\mu_1\mu_2 + \frac{N^2r^{(3)}}{(N-1)^{(2)}}\mu_1^3 \tag{4.09} $$

$$ \begin{aligned} {}_r\mu_4 = {} & \frac{r(N-r)}{(N-1)^{(3)}}\left[(N-2r)(N-3r)-N(r-1)\right]\mu_4 \\ & + \frac{4N(N-r)(N-2r+1)r^{(2)}}{(N-1)^{(3)}}\mu_1\mu_3 + \frac{6N^2r^{(3)}(N-r)}{(N-1)^{(3)}}\mu_1^2\mu_2 \\ & + \frac{N^3r^{(4)}}{(N-1)^{(3)}}\mu_1^4 + \frac{3N.r^{(2)}(N-r)(N-r-1)}{(N-1)^{(3)}}\mu_2^2 \end{aligned} \tag{4.10} $$

If we now convert these into the corresponding mean moments in the usual way, we obtain:

$$ {}_rm_2 = \frac{r(N-r)}{N-1}m_2 \tag{4.11} $$

$$ {}_rm_3 = \frac{r(N-r)(N-2r)}{(N-1)(N-2)}m_3 \tag{4.12} $$

$$ {}_rm_4 = \frac{r(N-r)[(N-2r)(N-3r)-N(r-1)]}{(N-1)(N-2)(N-3)}m_4 + \frac{3r^{(2)}(N-r)^{(2)}}{(N-1)^{(3)}}m_2^2 \tag{4.13} $$

Hence we may express the first and second *Pearson* coefficients, ${}_r\beta_1$ and ${}_r\beta_2$ of the r-fold sample distribution in terms of the corresponding coefficients of the unit sampling distribution as follows:

$$ {}_r\beta_1 = \frac{(N-1)(N-2r)^2}{r(N-r)(N-2)^2}\beta_1 \tag{4.14} $$

$$ \begin{aligned} {}_r\beta_2 = {} & \frac{(N-1)[(N-2r)(N-3r)-N(r-1)]}{r(N-r)(N-2)^{(2)}}\beta_2 \\ & + \frac{3N^{(2)}(N-r-1)(r-1)}{r(N-r)(N-2)^{(2)}} \end{aligned} \tag{4.15} $$

If N is so large that we can neglect $4N^{-1}$ we may simplify the above by the substitution of the sampling fraction $F = rN^{-1}$, *viz.*:

$$ {}_r\beta_1 = \frac{(1-2F)^2}{r(1-F)}\,\beta_1 \tag{4.16} $$

$$ {}_r\beta_2 = \frac{3(r-1)}{r} + \frac{1-6F(1-F)}{r(1-F)}\,\beta_2 \tag{4.17} $$

The hypergeometric distribution for sampling in the binary taxonomic domain is, of course, a particular case of the distribution whose first two β-coefficients are as defined by (4.14)–(4.15). For a universe with score classes 0 and 1, the respective frequencies being q and p the k^{th} moment of the u–s–d is $m_k = q(-p)^r + pq^r$, whence the first two β-coefficients of the unit sample distribution are:

$$ {}_r\beta_1 = \frac{(N-1)\,(N-2r)^2}{r(N-r)\,(N-2)^2} \cdot \frac{(q-p)^2}{p^2q^2} \tag{4.18} $$

$$ {}_r\beta_2 = \frac{(N-1)}{(N-2)\,(N-3)\,r\,(N-r)pq}\Big[N(N+1) - 6r(N-r) + 3pq \Big\{N^2(r-2) - Nr^2 + 6r(N-r)\Big\}\Big] \tag{4.19} $$

The relations defined by (4.14)–(4.15) are equally appropriate to the score sum and the mean score distribution and (4.18)–(4.19) to the raw score and the proportionate score. The k^{th} mean moment of the mean (or proportionate) score of the r-fold sample distribution is, of course, obtainable from the foregoing formulae for those of the score-sum (or raw score) by applying the appropriate scalar factor (r^{-k}). Comparison of (4.14)–(4.15) with (3.11)–(3.12) brings into focus fundamental differences between sampling with and without replacement from any finite universe. In either case the r-fold sample score distribution must be symmetrical if the unit sample distribution is also. In either case, the kurtosis is approximately normal $({}_r\beta_2 \simeq 3.0)$ if r is large and not more than $^4/_5{}^{ths}$ (*vide infra*) as large as N. Here the resemblance ends. If we sample without replacement ${}_r\beta_1$ and ${}_r\beta_2$ both become infinite when r = N, as we should expect since there is then only one sample score-sum with non-zero frequency.

Regardless of the structure of a skew finite universe sampling with replacement implies that the distribution becomes progressively less skew as the size of the sample increases; but ${}_r\beta_1$ vanishes if the sampling fraction is 0.5 when we sample without replacement.

This of itself, though a necessary, is not a sufficient condition of symmetry but the symmetry of the distribution when $F = \frac{1}{2}$ follows from elementary principles, if we bear in mind the fact that each combination of a out of N letters corresponds to the same number, i.e. $N^{(a)}$ of permutations. Hence the frequencies of score-sums will be in the same ratio as the numbers of combinations of items whose total score is the same. If $a = \frac{1}{2}N$, there will be a unique combination with score sum $S_N - s_a$ for each unique combination with score sum s_a. Thus scores of s_a and $S_N - s_a$ will occur with equal frequency when $a = \frac{1}{2}N$. The mean score sum is then $\frac{1}{2}S_N$ and the deviation of s_a therefrom is $(s_a - \frac{1}{2}S_N) = +S_a$. That of each corresponding combination whose score sum is $(S_N - s_a)$ will be $(S_N - s_a - \frac{1}{2}S_N) = (\frac{1}{2}S_N - s_a) = -S_a$. Thus score deviation of $+S_a$ and $-S_a$ must occur with equal frequency.

From (4.15) we see that ${}_r\beta_2$ must be less than 3.0, when $F = \frac{1}{2}$. at which level we may therefore expect Type II to give the best description of the sampling distribution if any continuous curve of the *Pearson* system is suitable. An examination of the approximate formula (4.17) brings into focus what is perhaps a more remarkable feature of nonreplacement sampling distributions than the symmetry of that of the half-universe sample. The expression $1-6F(1-F)$ vanishes when $F = \frac{1}{2} \pm {}^1/\sqrt{12}$, i.e. $F \simeq 0.22$ or 0.79 between which limits the coefficient of β_2 in the second term of (4.17) is negative with a numerical maximum for $F \simeq 0.59$. Regardless of the structure of the universe, the sampling distribution will always be platykurtic unless the sampling fraction in round numbers is greater than four-fifths; *ceteris paribus*, below this level a highly leptokurtic unit-sample distribution will generate a more platykurtic sampling distribution than a distribution which is initially flatter than the normal curve, e.g. a rectangular or indeed even a U-shaped one. At $F \sim 0.59$ the kurtosis is a minimum and (4.17) is approximately

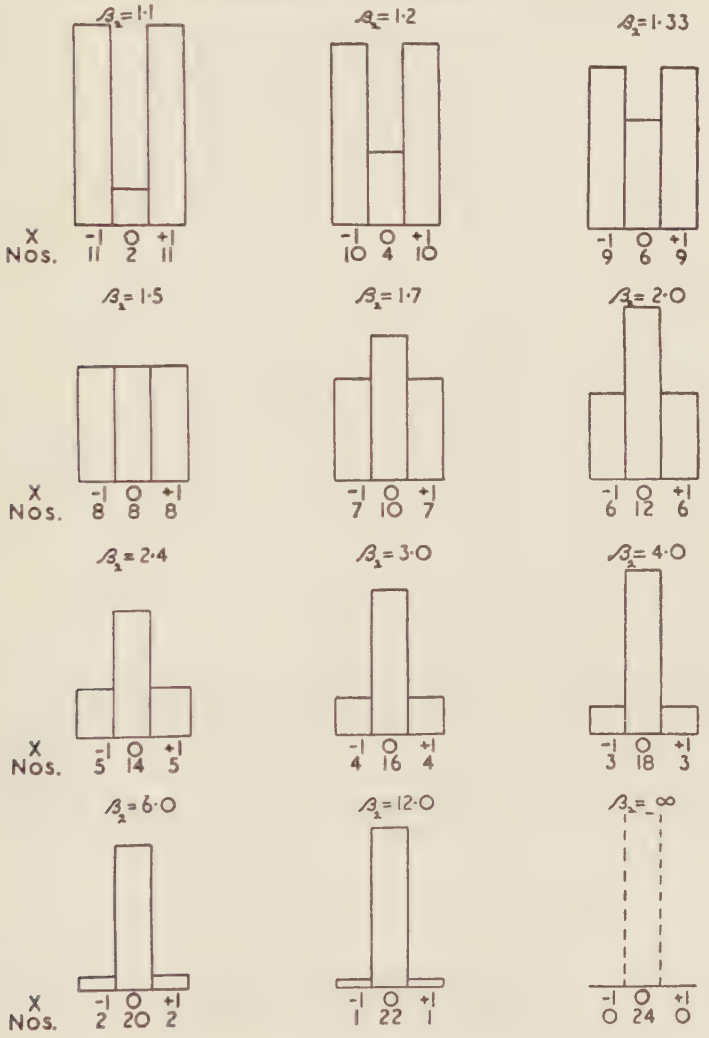
$$3 - \frac{3 + 1.1\ \beta_2}{r}$$

This seemingly paradoxical characteristic of any non-replacement distribution comes into focus (fig. 1) if we calculate from the exact formula (4.15) the kurtosis for samples of different sizes extracted without replacement from a symmetrical 24-fold universe of only 3 score classes. For simplicity, we may assign to the 3-classes scores of -1, 0 and $+1$, and frequencies (p_a, p_b, p_c) as below with β_2

values for the unit-sample distribution in the range 1–12 including a rectangular and a U-shaped contour at the lower limit of kurtosis.

−1	0	+1	β_2	−1	0	+1	β_2
1	22	1	12.0	5	14	5	2.4
2	20	2	6.0	6	12	6	2.0
3	18	3	4.0	8	8	8	1.5
4	16	4	3.0	11	2	11	1.1

Fig. 1. 24-fold Symmetrical Universes of 3 classes.



The picture disclosed by fig. 1 raises the question: what lower limit may ${}_r\beta_2$ attain if β_2 is as high as may be? This admits no simple answer because the size of the universe itself sets a limit both to the maximum value of β_2 and to the value of r consistent with the condition that the second term in (4.17) is both negative and numerically maximal, as when $F \simeq 0.6$. Thus a universe of 100 items assignable to 3 equally spaced score classes (—1, 0, +1) as above cannot have a kurtosis greater than 50, and the sample size consistent with a minimum value of ${}_r\beta_2$ is about 60.

For the rectangular universe $\beta_1 = 0$ and $\beta_2 \simeq 1.8$ when $n = N$ is large. The possibility of generating a rectangular sampling distribution when $F = 0.6$ therefore implies that $3+1.1\,\beta_2 = 1.2r$. If $N = 200$ and $F = 0.6$, $r = 120$, so that β_2 could satisfy this relation only if $\beta_2 \simeq 130$. For the 200-fold binomial universe defined by $(0.995 + 0.005)^{1}$, β_2 exceeds 130 but no other binomial 200-fold universe and no 200-fold universe of more than 2 non-zero classes can satisfy the condition $\beta_2 \geqslant 130$. From a 2-class universe of 1 zero score value and 199 unit score values the value of ${}_r\beta_2$ for the 120-fold sample would be 1.12; but the sample itself would contain only 2 score classes (*viz.* score sums of 120 and 119) as we see by expanding $(199 + 1)^{(120)}$. Though ${}_r\beta_2$ is in this case less than 1.8; the distribution of the sample score is therefore monotonic.

From (4.14)—(4.15) we see that the first two *Pearson* coefficients of the r-fold and the (N—r)-fold sample are respectively identical. Thus ${}_r\beta_1 = \beta_1$ and ${}_r\beta_2 = \beta_2$ when $r = (N—1)$. If β_2 lies in the neighbourhood of $3(N—1) \div (N+1)$ the kurtosis of the r-fold distribution does not appreciably change within the range $r = 1$ to $r = N—1$; e.g. when $N = 24$ and $\beta_2 = 2.76$ (see fig. 2).

For reasons which will be apparent later, in this context we may speak of a universe as a large one if $N \geqslant 100$ and $n \geqslant 7$. Of such a universe we may then say that ${}_r\beta_1$ necessarily has the same value as the normal distribution when $F = \frac{1}{2}$ and ${}_r\beta_2$ is nearly 3 when $F \simeq 0.2$. It is therefore of interest to exhibit the magnitudes of ${}_r\beta_1$ and ${}_r\beta_2$ when the sampling fraction lies midway between these two limits *viz.* $F = {}^{7}/_{20}$. We then derive from 4.16 and 4.17:

$$ {}_r\beta_1 \simeq \frac{9}{65r} \cdot \beta_1 \quad ; \quad {}_r\beta_2 \simeq 3 - \frac{73}{130r} \cdot \beta_2 $$

If $N = 100$, $r = 35$ for this value of F, in which case the numerical values of the above are $0.004\,\beta_1$ and $3—0.016\,\beta_2$.

It is clear from this that the *Pearson* coefficients of a one-third non-replacement sample from a sizeable universe will differ very little from their normal values. However the suitability of any continuous curve as a descriptive function of a discrete sampling distribution will be limited by the number of r-fold sample score classes and these may diminish with the size of sample if there is no replacement. We shall refer to this restriction more explicitly at a later stage. Meanwhile it is of interest to examine the implications of the expressions derived above when the sampling fraction is

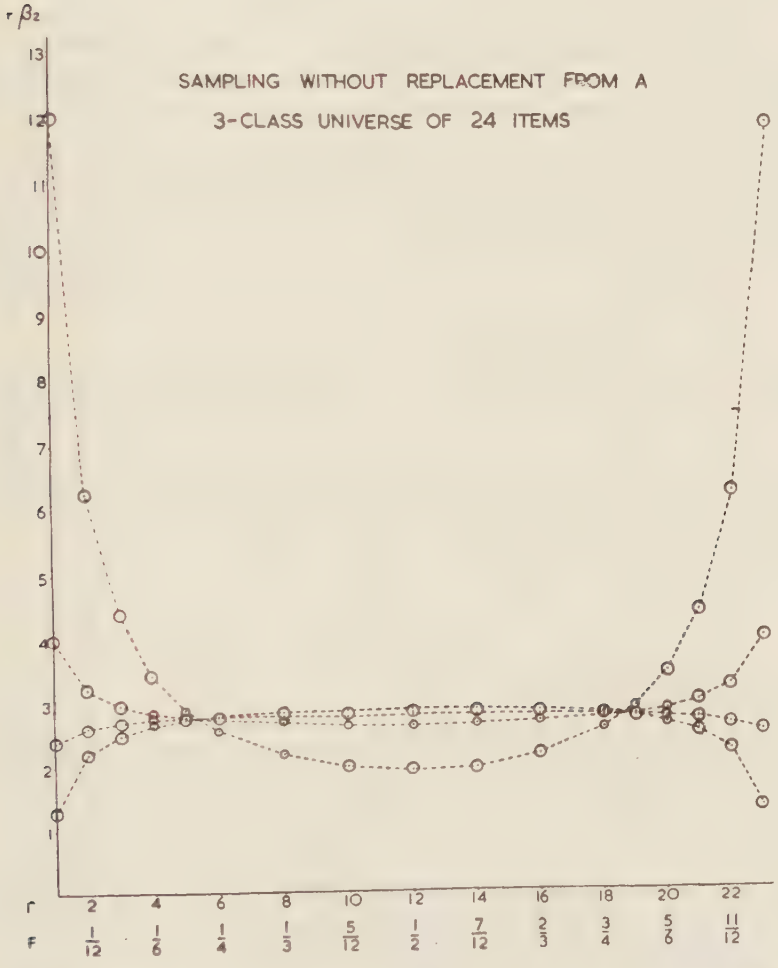


Fig. 2. Variation of Kurtosis Coefficient ($_r\beta_2$) with size of sampling fraction (F).

much smaller e.g. $F = {}^1/_6$. We then find that ${}_r\beta_1 \simeq \frac{8}{15\,r} \cdot \beta_1$ and ${}_r\beta_2 \simeq 3 + \frac{1}{5\,r} \cdot \beta_2$. In this case for $N = 100$, $r \simeq 17$ and then ${}_r\beta_1 \simeq 0.03$. β_1 and ${}_r\beta_2 \simeq 3 + 0.01\,\beta_2$. For values of β_2 in the range 2 to 5 and of β_1 in the range 2/3 to 5/3 the values of ${}_r\beta_1$ and ${}_r\beta_2$ correspond to the *Poisson* values in the range $M = 50$ to 20. Actually the normal distribution tallies closely with the *Poisson* at the $2\,\sigma$ level when $M \geqslant 10$.

5. Non-replacement score difference distributions.

When we turn to the distribution of the difference between the a-fold and the b-fold score sum, the distinction mentioned at the end of § 4 becomes important; and it will be the topic of further consideration in the second communication of this series. If our sample score is the score-sum (i.e. the raw score in the binary taxonomic domain) we may write the difference in accordance with the notation of § 2 as

$$\begin{aligned} {}_{a-b}X_{\cdot} &= {}_aX_{\cdot} - {}_bX_a \\ &= x_1 + x_2 \ldots + x_a - x_{a+1} - x_{a+2} \ldots - x_{a+b} \end{aligned}$$

Hence for the k^{th} zero moment of the difference distribution we have:

$${}_{a-b}\mu_k = E(x_1 + x_2 \ldots + x_a - x_{a-1} - x_{a-2} \ldots - x_{a+b})^k \qquad (5.01)$$

If our sample score is the mean-score (i.e. proportionate score in the binary taxonomic domain), the difference (d) is:

$$\begin{aligned} d &= \frac{{}^aX_{\cdot}}{a} - \frac{{}^bX_a}{b} \\ &= \frac{1}{ab}\,(b.x_1 + b.x_2 \ldots + b.x_a - a.x_{a+1} - a.x_{a+2} \ldots - a.x_{a+b}) \end{aligned}$$

If we denote the k^{th} zero moment of the mean score difference by ${}_d\mu_k$:

$${}_d\mu_k = \frac{1}{a^k b^k}\,E(bx_1 + bx_2 \ldots + bx_a - ax_{a+1} - ax_{a+2} \ldots - ax_{a+b})^k \qquad (5.02)$$

Each of the foregoing expressions (5.01) and (5.02) invites examination of three cases when the universe is both finite and discrete:

(a) sampling without replacement from one and the same universe;

(b) extraction without replacement of the a-fold and b-fold samples from identical different universes;

(c) sampling with replacement either from one and the same universe or from 2 identical universes.

The last case is formally the same as when we sample with or without replacement from a discrete infinite universe. The distinction between extracting the two samples from 2 identical universes or from one and the same universe being then irrelevant, since the two samples are *ex hypothesi* independent in either case. We shall defer more detailed consideration of (c) to subsequent communications concerned with sampling from the discrete universe, since it is easy to derive by the iterative method of § 3 expressions for the first six *Pearson* coefficients in the same form as (3.11)—(3.16). Here it suffices to say that (c) implies:

$$\mathrm{E}(\mathrm{x}_u^a.\mathrm{x}_v^b.\mathrm{x}_w^c\ldots) = \mu_{a.b.c..} = \mu_a.\mu_b.\mu_c.. \tag{5.03}$$

The expansion of (5.01) then proceeds as for the derivation of (3.21)—(3.23) from (3.17) if we substitute the H-coefficients of (3.24) for the C-coefficients of (3.20), e.g.

$$_{(a—b)}\mu_2 = \mathrm{H}_{2.0}\mu_2 + \mathrm{H}_{1.1}\mu_1^2$$

Thus we derive at one step:

$$\left.\begin{aligned}
_{(a—b)}\mu_1 &= (a—b)\,\mu_1 \\
_{(a—b)}\mu_2 &= (a+b)\,\mu_2 + (a—b)^{[2]}\mu_1^2 \\
_{(a—b)}\mu_3 &= (a—b)\,\mu_3 + 3(a^{(2)}—b^{(2)})\,\mu_2.\,\mu_1 + (a—b)^{[3]}\mu_1^3 \\
_{(a—b)}\mu_4 &= (a+b)\,\mu_4 + 4(a—b)^{[2]}\mu_3.\,\mu_1 + 3(a+b)^{(2)}\mu_2^2 + \\
&\quad 6(a^{(3)}—ba^{(2)}—ab^{(2)}+b^{(3)})\,\mu_2.\,\mu_1^2 + (a—b)^{[4]}\mu_1^4
\end{aligned}\right\} \tag{5.04}$$

For the development of (5.02) on the same assumption, i.e. sampling with replacement in accordance with (5.03), we need expressions analogous to the H-coefficients of (3.20) when each unit sample score is a product of which one factor is either b or — a. We then write (5.02) in the form:

$$_d\mu_2 = \frac{1}{a^2b^2}\left[\mathrm{P}_{2.0}\mu_2 + \mathrm{P}_{1.1}\mu_1^2\right] \text{ etc.}$$

For the derivation of the first 4 zero moments of the proportion-

ate score difference on the replacement assumption or the assumption that N is indefinitely large, it will suffice to cite:

$$\left.\begin{aligned}
P_{2.0} &= ab\,(a+b)\\
P_{1.1} &= -ab\,(a+b)\\
P_{3.0} &= ab\,(b^2-a^2)\\
P_{2.1} &= -3ab\,(b^2-a^2)\\
P_{1.1.1} &= 2ab\,(b^2-a^2)\\
P_{4.0} &= ab\,(a^3+b^3)\\
P_{3.1} &= -4ab\,(a^3+b^3)\\
P_{2.2} &= 3ab\left\{ab\,(a^2+b^2)-(a^3+b^3)+2a^2b^2\right\}\\
P_{2.1.1} &= 6ab\left\{2\,(a^3+b^3)-ab\,(b^2+a^2)-2a^2b^2\right\}\\
P_{1.1.1.1} &= 3ab\left\{ab\,(a^2+b^2)+2a^2b^2-2\,(a^3+b^3)\right\}
\end{aligned}\right\} \qquad (5.05)$$

If we sample *without replacement from one and the same universe*, we may derive the moments of the difference distribution of the score-sum and of the mean score in the same way except insofar as we interpret the co-moments $\mu_{1.1}$, $\mu_{2.1}$ etc. as in § 4. We thus obtain for the score-sum (or raw score) difference distribution:

$$_{(a-b)}\mu_2 = [\{(a+b)N-(a-b)^2\}N\mu_2 + N^2(a-b)^{[2]}\mu_1^2] \div N^{(2)} \qquad (5.06)$$

$$\begin{aligned}
_{(a-b)}\mu_3 = (a-b)\,[&\{N^2-3N\,(a+b)+2\,(a-b)^2\}N\mu_3\\
&+3N^2\{N\,(a+b-1)-(a-b)^2+(a+b)\}\mu_2\mu_1\\
&+N^3(a-b)^{[3]}\mu_1^3] \div N^{(3)}
\end{aligned} \qquad (5.07)$$

$$\begin{aligned}
_{(a-b)}\mu_4 = &\left[(a+b)-\frac{\{4\,(a-b)^{[2]}+3\,(a+b)^{(2)}\}}{N-1}\right.\\
&\left.+\frac{12\,(a^{(3)}-ba^{(2)}-ab^{(2)}+b^{(3)})}{(N-1)^{(2)}}-\frac{6\,(a-b)^{[4]}}{(N-1)^{(3)}}\right]\mu_4\\
&+\frac{4N}{(N-1)}\left[(a-b)^{[2]}-\frac{3(a^{(3)}-ba^{(2)}-ab^{(2)}+b^{(3)})}{(N-2)}\right.
\end{aligned}$$

$$+\frac{2(a-b)^{[4]}}{(N-2)^{(2)}}\Big]\mu_1\mu_3$$

$$+\frac{6N^2}{(N-1)^{(2)}}\left[(a^{(3)}-ba^{(2)}-ab^{(2)}+b^{(3)})-\frac{(a-b)^{[4]}}{(N-3)}\right]\mu_1^2\mu_2$$

$$+\frac{3N}{N-1}\left[(a+b)^{(2)}-\frac{2(a^{(3)}-ba^{(2)}-ab^{(2)}+b^{(3)})}{(N-2)}\right.$$

$$\left.+\frac{(a+b)^{[4]}}{(N-2)^{(2)}}\right]\mu_2^2+\frac{N^3(a-b)^{[4]}}{(N-1)^{(3)}}\mu_1^4 \qquad (5.08)$$

The corresponding *mean* moments are:

$${}_{(a-b)}m_2=\frac{(a+b)N-(a-b)^2}{N-1}m_2$$

$${}_{(a-b)}m_3=\frac{(a-b)}{(N-1)^{(2)}}\{N^2-3N(a+b)+2(a-b)^2\}m_3 \qquad (5.10)$$

$${}_{(a-b)}m_4=\left[(a+b)(N-\overline{a+b})\{N^2-6N(a+b)+6(a+b)^2+N\}\right.$$

$$\left.+16ab\{N^2-3N(a+b)+3(a^2+b^2)+N\}\right]\frac{m_4}{(N-1)^{(3)}}$$

$$+\frac{3N}{(N-1)^{(3)}}[(a+b)^{(2)}(N-\overline{a+b})^{(2)}+8ab\{(a+b)N-(a^2+b^2)$$

$$-2(N-1)\}]m_2^2 \qquad (5.11)$$

Hence for the score-sum difference distribution on the assumption that sampling is without replacement from a single universe:

$${}_{(a-b)}\beta_1=\frac{(a-b)^2\{N^2-3N(a+b)+2(a-b)^2\}^2(N-1)}{\{N(a+b)-(a-b)^2\}^3(N-2)^2}\beta_1 \qquad (5.12)$$

$${}_{(a-b)}\beta_2=$$

$$\frac{(N-1)[s(N-s)\{N(N+1)-6s(N-s)\}+16ab\{N(N+1)-3s(N-s)-6ab\}]}{(N-2)^{(2)}\{s(N-s)+4ab\}^2}\beta_2$$

$$+\frac{3N^{(2)}[s^{(2)}(N-s)^{(2)}+8ab\{s(N-s)+2(ab-N+1)\}]}{(N-2)^{(2)}\{s(N-s)+4ab\}^2} \qquad (5.13)$$

in which $s=a+b$

For the 2-class universe in the customary symbolism:

$$_{(a-b)}\beta_1 = \frac{(a-b)^2(q-p)^2(N-1)\{N^2-3N(a+b)+2(a-b)^2\}^2}{pq(N-2)^2\{N(a+b)-(a-b)^2\}^3} \tag{5.14}$$

$$_{(a-b)}\beta_2 = \frac{(N-1)}{(N-2)^{(2)}pq} \cdot$$
$$\frac{\left[s(N-s)\{N(N+1)-6s(N-s)\}+16ab\{N(N+1)-3s(N-s)-6ab\}\right]}{\{s(N-s)+4ab\}^2}$$
$$+\frac{3(N-1)}{(N-2)^{(2)}}\left[(N+6)-\frac{N^2}{\{s(N-s)+4ab\}}-\frac{28ab.N^2}{\{s(N-s)+4ab\}^2}\right] \tag{5.15}$$

When we sample without replacement from 2 identical universes, or replace the first (a-fold) non-replacement sample before extracting without replacement the second (b-fold) one, it is simplest to develop the moments of the two types of difference distribution as follows:

Score-sums:

$$_{a-b}\mu_k = E(_ax_. - {}_bx_.)^k = \sum_{w=0}^{w=k} (-1)^w k_{(w)}\, E(_ax_.)^w(_bx_.)^{k-w}$$

$$\therefore \quad _{a-b}\mu_k = \sum_{w=0}^{w=k} (-1)^w\, k_{(w)}\, {}_a\mu_w \cdot {}_b\mu_{k-w} \tag{5.16}$$

Mean Scores:

$$_d\mu_k = E(\frac{{}^ax_.}{a} - \frac{{}^bx_.}{b})^k = \frac{1}{a^kb^k}\, E\,(b_ax_. \quad a_bx_.)^k$$

$$\therefore \quad _d\mu_k = \frac{1}{a^kb^k} \sum_{w=0}^{w=k} (-1)^w\, b^w.a^{k-w}\, {}_a\mu_w \cdot {}_b\mu_{k-w} \tag{5.17}$$

The appropriate expressions for $_a\mu_w$ or $_b\mu_{k-w}$ in the above are, of course, as developed in (4.08) – (4.10) above.

From (5.16) in this way we derive

$$_{a-b}\beta_1 = \frac{(N-1)(a-b)^2\left[(N-s)(N-2s)-2ab\right]^2}{(N-2)^2\left[s(N-s)+2ab\right]^3}\,\beta_1 \tag{5.18}$$

$$_{a-b}\beta_2 = \frac{N-1}{(N-2)^{(2)}}\,\frac{(N-s)(N-2s)\{s(N-3s)+12ab\}+2ab\{N(N+1)-6ab\}}{\{s(N-s)+2ab\}^2}\,\beta_2$$
$$+\frac{3N^{(2)}}{(N-2)^{(2)}}\,\frac{\left[s^{(2)}(N-s)^{(2)}-2ab\{(N-s)(N-2s)+N-ab-1\}\right]}{\{s(N-s)+2ab\}^2}$$

$$+\frac{6ab(N-a)(N-b)}{\{s(N-s)+2ab\}^2} \tag{5.19}$$

in which $s = a+b$

Similarly, we derive from (5.17):

$${}_d\beta_1 = (\frac{1}{t}-\frac{1}{s})^2\frac{N^2(N-1)(NR-3)^2}{(N-2)^2(NR-2)^3}\beta_1 \tag{5.20}$$

$$\begin{aligned}{}_d\beta_2 = {} & \frac{(N-1)(RN-1)}{(N-2)^{(2)}}\left[\frac{RN-3}{RN-2}-\frac{RN(1-R)}{(RN-2)^2}\right]\beta_2 \\ & -\frac{(N-1)}{(N-2)^{(2)}}\frac{[3N^2(N+1)R-2N(7N+1)+6ab]}{ab(NR-2)^2}\beta_2 \\ & +\frac{3N^{(2)}}{(RN-2)^2(N-2)^{(2)}}\Big[(1-R)(NR-R-1)(NR-1)+1 \\ & \qquad +\frac{1}{ab}\{3RN(N-1)-2(N^2+N-1)\}\Big] \\ & +\frac{6\{N^2-ab(NR-1)\}}{ab(NR-2)^2}\end{aligned} \tag{5.21}$$

in which $R = \frac{1}{a}+\frac{1}{b}$

The foregoing expressions like those for the sample mean in § 4 above give us some insight into the type of curve likely to be satisfactory for purposes of quadrature. It will suffice to comment from this viewpoint on the expressions for the first two *Pearson* coefficients of the difference distribution defined by (5.12) and (5.13) for the situation in which both samples come from the same finite universe. Both expressions simplify greatly, if we choose samples of equal size ($a=b$) in which event ${}_{(a-b)}\beta_1 = 0$ and ${}_{(a-b)}\beta_2$ reduces to:

$$\begin{aligned}& \frac{(N-1)}{2aN(N-2)^{(2)}}\{N(N-6a)+N+6a\}\beta_2 \\ & \quad +\frac{3(N-1)}{2aN(N-2)^{(2)}}\{(2a-1)(N-2a)^{(2)}+8a^2(N-a)-8a(N-1)\}\end{aligned}$$

If $a = N(N+1) \div 6(N-1) = b$, it is thus apparent that the difference distribution is symmetrical; and the value of the second *Pearson* Coefficient is independent of β_2, i.e. of the structure of universe. On substitution of this sample size in the expression above we find that ${}_{(a-b)}\beta_2$ reduces to $3(N-1) \div (N+1)$; but the interpretation of this result is so meaningful only within the framework of the assumption that both N and a must be integers. Evidently the coefficient of β_2 will be small if $N = 6a$, i.e. *each* sample is a one sixth fraction of the universe of choice. For larger values of N we may thus say that an *overall* sampling fraction of one third will ensure that the kurtosis of the difference distribution is independent of the kurtosis of the u–s–d. More generally for $N = 6a$, ${}_{(a-b)}\beta_2$ reduces to:

$$\frac{6(N-1)^2}{N^{(4)}} \beta_2 + \frac{3(N-1)^2(N^2-6N+6)}{N^{(4)}}$$

The maximum finite value of β_2 occurs in the binary universe, the frequencies of the classes being N^{-1} and $(N-1)N^{-1}$ respectively, one class being then represented by only one member. The second *Pearson* Coefficient of its u–s–d is $(N^2-3N+3) \div (N-1)$. This is its maximum value; and the maximum value of ${}_{(a-b)}\beta_2$ is therefore exactly 3. Thus the difference distribution is necessarily platykurtic and the greatest contribution which can be made by the term involving β_2 is $6(N^2-3N+3) \div N(N-2)^{(2)}$. The table below shows for various values of N, the values of the two terms in ${}_{(a-b)}\beta_2$ calculated on the assumption that β_2 has its maximum value, as above.

N	1st term	2nd term	N	1st term	2nd term
6	1.75	1.25	30	0.21	2.79
12	0.62	2.38	42	0.15	2.85
18	0.38	2.62	60	0.10	2.90
24	0.27	2.73	96	0.06	2.94

Even if the u–s–d of the binary universe is very platykurtic we therefore see that samples of size equal to $^1/_6$th of the universe will generate a symmetrical difference distribution having a second *Pearson* Coefficient greater than or equal to 2.8 if N is greater than or equal to 30. For any universe of 30 or more items, regardless of the number of score classes and of items in each, there is good reason

to assume that the first two *Pearson* coefficients of the distribution of the difference referable to equal samples of one sixth will lie very close to their normal values. However, this does not suffice to justify the conclusion that the normal curve will give an adequate quadrature for the sample difference distribution. An examination of how gratuitous such an assumption may be will indeed give us some insight into circumstances which guarantee a good fit.

In particular, we recall the case of sampling from a 2-class universe. Without restriction on the values of a and b, the difference distribution is then definable as follows for a u–s–d of score values differing by unit increment:

Difference Scores	—1	0	+1
Frequencies	$\frac{a}{N}$	$\frac{N-a-b}{N}$	$\frac{b}{N}$

When $a = b$ and $F = (a+b) \div N$ is the total sampling fraction, this reduces to:

Difference Scores	—1	0	+1
Frequencies	$\frac{F}{2}$	1—F	$\frac{F}{2}$

Whence ${}_{(a-b)}\beta_1 = 0$ and ${}_{(a-b)}\beta_2 = 3.0$ if $a = b$ and $F = \frac{1}{3}$.

The difference distribution is then a special case of what we call in a subsequent contribution of this series the *burette universe, viz.*:

Score	—1	0	+1
Frequency	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

We may here make use of results obtainable from sampling in the burette (infinite discrete 3-class) universe by stating at this stage without proof the following conclusion: when the first two *Pearson* cofficients of a distribution involving 20 score classes are very close to their normal values, we may confidently invoke the normal distribution for purposes of quadrature adequate for statistical usage. It is therefore immaterial to examine the implications of the foregoing formulae for ${}_{(a-b)}\beta_1$ and ${}_{(a-b)}\beta_2$ more closely. It suffices to state of any finite unimodal universe that:

(a) the first 2 coefficients of the non-replacement difference distribution w.r.t. a-fold and b-fold samples will lie very close to their normal values if both the following conditions hold good: (i) the sample sizes are equal (a = b); (ii) the total sampling fraction (F = (a+b) ÷ N) is in the neighbourhood of one third;

(b) the normal curve will then give a satisfactory quadrature if the distribution of the a-fold sample is referable to at least 10 different score values.

6. Numerical illustrations of sampling in the finite universe.

In § 5 above, we have intimated that two conditions are pre-eminently relevant to the adequacy of the normal as a descriptive curve for approximate quadrature of a unimodal r-fold sampling distribution, *viz.*: (i) how closely the *Pearson* coefficients of symmetry and kurtosis approximate to their normal values of 0 and 3 respectively; (ii) how many (n_r) score classes are specifiable by frequencies other than zero. To give due weight to both considerations last stated we need to recall one conclusion already stated and a second implicit in the non-replacement condition:

(a) the condition that ${}_r\beta_2$ is near 3.0 is that the sampling fraction F is in the neighbourhood of 0.35; and that ${}_r\beta_1$ is near zero is that $F \simeq 0.5$.

(b) the condition that n_r is a maximum is that the u–s–d is rectangular or U-shaped to ensure the minimum number of vanishing terms in the hypergeometric series for the r-fold sample distribution, whence $\beta_2 < 1.8$.

Having regard to these conditions, we here present tables of the distribution of samples of 14 (F = 0.35) and 20 (F = 0.5) taken from 5-class 40-fold universes (A–F) as follows

Ref. No.	Score values of u.s.d. −2	−1	0	+1	+2	β_1	β_2
A	14	4	4	4	14	0	1.27
B	8	8	8	8	8	0	1.70
C	5	8	14	8	5	0	2.24
D	2	6	24	6	2	0	3.88
E	12	10	8	6	4	0.206	2.03
F	2	4	20	10	4	0.025	3.23

Tables I-VIII exhibit the appropriate r-fold ($r = 14$ or $r = 20$) sample distributions with corresponding values of ${}_r\beta_1$ and ${}_r\beta_2$ for the above in juxtaposition to the normal distribution for unit variance with due regard to the appropriate half-interval correction.

We may get the salient features of these tables into focus if we now tabulate side by side the area of the tail of the normal distribution cut off at values near the 2σ (so-called significance) level and the exact sum of frequency terms excluded. The results show a remarkably close fit for a wide range of 40-fold universes.

Significance Level	Size of Sample	Pearson Coefficients of the u-s-d β_1	β_2	Excluded area of Normal	Sum of excluded frequencies
2.0788	14	0	1.27	0.01426	0.01488
2.0831	14	0	1.70	0.01314	0.01395
1.9360	14	0	2.24	0.01800	0.01903
1.9486	14	0	3.88	0.01388	0.01604
1.9795	14	0.206	2.03	0.00903	0.00886
1.9110	14	0.025	3.23	0.00828	0.00927
1.8883	20	0.206	2.03	0.02186	0.02241
1.9887	20	0.025	3.23	0.01410	0.01561

Table I. Comparison between the Normal Integral and a 14-fold non-replacement sample drawn from Universe A.

$\sigma = 5.2916$ ${}_r\beta_1 = 0$ ${}_r\beta_2 = 2.92$

Standard Score Deviation $\frac{X}{\sigma}$	Frequency		Cumulative frequency	
	Exact	Normal	Exact	Normal
0	0.07461	0.07539	0.07461	0.07528
0.1890	0.07337	0.07405	0.22135	0.22320
0.3780	0.06973	0.07019	0.36081	0.36341
0.5669	0.06392	0.06420	0.48865	0.49170
0.7559	0.05664	0.05665	0.60193	0.60490
0.9449	0.04850	0.04824	0.69893	0.70115
1.1339	0.04006	0.03964	0.77905	0.78068
1.3228	0.03188	0.03143	0.84281	0.84360
1.5118	0.02449	0.02404	0.89179	0.89179
1.7008	0.01813	0.01775	0.92805	0.92739
1.8898	0.01290	0.01264	0.95385	0.95277
2.0788	0.00881	0.00869	0.97147	0.97024

$\sigma = 5.2916$		$_r\beta_1 = 0$	$_r\beta_2 = 2.92$	
Standard Score Deviation $\frac{X}{\sigma}$	Frequency		Cumulative frequency	
	Exact	Normal	Exact	Normal
2.2677	0.00580	0.00576	0.98307	0.98295
2.4567	0.00365	0.00369	0.99037	0.98926
2.6457	0.00220	0.00228	0.99477	0.99384
2.8347	0.00126	0.00136	0.99729	0.99659
3.0237	0.00069	0.00078	0.99867	0.99817
3.2126	0.00036	0.00043	0.99939	0.99906
3.4016	0.00018	0.00023	0.99975	0.99953
3.5906	0.00008	0.00012	0.99991	0.99976
3.7796	0.00004	0.00006	0.99998	0.99989
3.9686	0.00001	0.00003	1.00000	0.99996

Table II. Comparison between the Normal Integral and a 14-fold non-replacement sample drawn from Universe B.

$\sigma = 4.3205$		$_r\beta_1 = 0$	$_r\beta_2 = 2.90$	
Standard Score Deviation $\frac{X}{\sigma}$	Frequency		Cumulative frequency	
	Exact	Normal	Exact	Normal
0	0.09120	0.09234	0.09120	0.09217
0.2314	0.08890	0.08989	0.26900	0.27158
0.4619	0.08234	0.08299	0.43368	0.43718
0.6944	0.07244	0.07255	0.57856	0.58211
0.9258	0.06049	0.06015	0.69954	0.70236
1.1573	0.04790	0.04726	0.79534	0.79698
1.3887	0.03592	0.03521	0.86718	0.86753
1.6202	0.02548	0.02485	0.91814	0.91740
1.8516	0.01705	0.01663	0.95224	0.95084
2.0831	0.01074	0.01054	0.97372	0.97211
2.3145	0.00635	0.00634	0.98642	0.98490
2.5460	0.00351	0.00361	0.99344	0.99222
2.7775	0.00181	0.00195	0.99706	0.99618
3.0089	0.00086	0.00100	0.99878	0.99822
3.2404	0.00038	0.00048	0.99954	0.99921
3.4719	0.00015	0.00022	0.99984	0.99966
3.7033	0.00005	0.00009	0.99994	0.99987
3.9347	0.00002	0.00004	0.99998	0.99995
4.1662	0.00001	0.00002	1.00000	0.99998

Table III. Comparison between the Normal Integral and a 14-fold non-replacement sample drawn from Universe C.

$\sigma = 3.6157$ $\quad {}_r\beta_1 = 0$ $\quad {}_r\beta_2 = 2.92$

Standard Score Deviation $\frac{X}{\sigma}$	Frequency		Cumulative frequency	
	Exact	Normal	Exact	Normal
0	0.10867	0.11033	0.10867	0.11005
0.2766	0.10483	0.10619	0.31833	0.32177
0.5531	0.09407	0.09468	0.50647	0.51072
0.8297	0.07845	0.07819	0.66337	0.66695
1.1063	0.06070	0.05983	0.78477	0.78670
1.3829	0.04347	0.04241	0.87171	0.87176
1.6594	0.02872	0.02785	0.92915	0.92777
1.9360	0.01743	0.01694	0.96401	0.96194
2.2126	0.00966	0.00954	0.98333	0.98125
2.4891	0.00485	0.00498	0.99303	0.99139
2.7657	0.00219	0.00241	0.99741	0.99631
3.0423	0.00087	0.00108	0.99915	0.99852
3.3189	0.00030	0.00045	0.99975	0.99945
3.5954	0.00009	0.00017	0.99993	0.99981
3.8720	0.00002	0.00006	0.99997	0.99994

Table IV. Comparison between the Normal Integral and a 14-fold non-replacement sample drawn from Universe D.

$\sigma = 2.5660$ $\quad {}_r\beta_1 = 0$ $\quad {}_r\beta_2 = 2.81$

Standard Score Deviation $\frac{X}{\sigma}$	Frequency		Cumulative frequency	
	Exact	Normal	Exact	Normal
0	0.15228	0.15547	0.15228	0.15450
0.3897	0.14205	0.14410	0.43638	0.44120
0.7794	0.11512	0.11474	0.66662	0.67008
1.1691	0.08055	0.07849	0.82772	0.82741
1.5588	0.04813	0.04613	0.92398	0.92050
1.9486	0.02413	0.02329	0.97224	0.96791
2.3383	0.00989	0.01010	0.99202	0.98869
2.7280	0.00316	0.00377	0.99834	0.99652
3.1177	0.00072	0.00121	0.99978	0.99908
3.5074	0.00010	0.00033	0.99998	0.99968
3.8971	0.00001	0.00008	1.00000	0.99996

Table V. Comparison between the Normal Integral and a 14-fold non-replacement sample drawn from Universe E.

$\sigma = 4.0414$ $\quad {}_r\beta_1 = 0.002$ $\quad {}_r\beta_2 = 2.89$

Standard Score Deviation $\frac{X}{\sigma}$	Frequency		Cumulative frequency	
	Exact	Normal	Exact	Normal
—3.9590	0.00001	0.00004	0.00001	0.00006
—3.7116	0.00003	0.00010	0.00004	0.00017
—3.4641	0.00010	0.00024	0.00014	0.00042
—3.2167	0.00031	0.00056	0.00045	0.00099
—2.9693	0.00085	0.00120	0.00130	0.00223
—2.7218	0.00199	0.00243	0.00329	0.00469
—2.4744	0.00421	0.00462	0.00750	0.00938
—2.2269	0.00804	0.00827	0.01554	0.01772
—1.9795	0.01408	0.01392	0.02962	0.03175
—1.7321	0.02272	0.02202	0.05234	0.05389
—1.4846	0.03403	0.03279	0.08637	0.08678
—1.2372	0.04749	0.04592	0.13386	0.13275
—0.9898	0.06202	0.06048	0.19588	0.19323
—0.7423	0.07599	0.07493	0.27187	0.26808
—0.4949	0.08755	0.08734	0.35942	0.35525
—0.2474	0.09502	0.09549	0.45444	0.45075
0.0000	0.09727	0.09871	0.55171	0.54925
+0.2474	0.09398	0.09549	0.64569	0.64475
+0.4949	0.08574	0.08734	0.73143	0.73192
+0.7423	0.07387	0.07493	0.80530	0.80677
+0.9898	0.06008	0.06048	0.86538	0.86725
+1.2372	0.04610	0.04592	0.91148	0.91322
+1.4846	0.03332	0.03279	0.94480	0.94611
+1.7321	0.02267	0.02202	0.96747	0.96825
+1.9795	0.01447	0.01392	0.98194	0.98228
+2.2269	0.00866	0.00827	0.99060	0.99062
+2.4744	0.00483	0.00462	0.99543	0.99531
+2.7218	0.00251	0.00243	0.99794	0.99777
+2.9693	0.00120	0.00120	0.99914	0.99901
+3.2167	0.00053	0.00056	0.99967	0.99958
+3.4641	0.00021	0.00024	0.99988	0.99983
+3.7116	0.00008	0.00010	0.99996	0.99994
+3.9590	0.00002	0.00004	0.99998	0.99998
+4.2065	0.00001	0.00002	0.99999	0.99999

Table VI. Comparison between the Normal Integral and a 14-fold non-replacement sample drawn from Universe A.

$\sigma = 2.8781$ $\quad {}_r\beta_1 = 0.0003$ $\quad {}_r\beta_2 = 2.84$

Standard Score Deviation $\frac{X}{\sigma}$	Frequency		Cumulative frequency	
	Exact	Normal	Exact	Normal
—3.9957	0.00001	0.00004	0.00001	0.00007
—3.6482	0.00006	0.00018	0.00007	0.00026
—3.3008	0.00037	0.00060	0.00044	0.00089
—2.9533	0.00147	0.00177	0.00191	0.00273
—2.6059	0.00449	0.00463	0.00640	0.00751
—2.2584	0.01116	0.01013	0.01756	0.01855
—1.9110	0.02342	0.02229	0.04098	0.04118
—1.5635	0.04246	0.04083	0.08344	0.08230
—1.2161	0.06749	0.06617	0.15093	0.14863
—0.8686	0.09499	0.09505	0.24592	0.24356
—0.5212	0.11913	0.12100	0.36505	0.36415
—0.1737	0.13358	0.13653	0.49863	0.50000
+0.1737	0.13408	0.13653	0.63271	0.63585
+0.5212	0.12037	0.12100	0.75308	0.75644
+0.8686	0.09633	0.09505	0.84941	0.85137
+1.2161	0.06833	0.06617	0.91774	0.91770
+1.5635	0.04261	0.04083	0.96035	0.95882
+1.9110	0.02309	0.02229	0.98344	0.98145
+2.2584	0.01071	0.01013	0.99415	0.99249
+2.6059	0.00416	0.00463	0.99831	0.99727
+2.9533	0.00131	0.00177	0.99962	0.99911
+3.3008	0.00032	0.00060	0.99994	0.99974
+3.6482	0.00005	0.00018	0.99999	0.99993
+3.9957	0.00001	0.00004	1.00000	0.99999

Table VII. Comparison between the Normal Integral and a 20-fold non-replacement sample drawn from Universe E.

$\sigma = 4.2366$ $\quad {}_r\beta_1 = 0$ $\quad {}_r\beta_2 = 2.897$

Standard Score Deviation $\frac{X}{\sigma}$	Frequency		Cumulative frequency	
	Exact	Normal	Exact	Normal
0	0.09295	0.09416	0.09295	0.09298
0.2360	0.09052	0.09157	0.27399	0.27674
0.4721	0.08360	0.08423	0.44119	0.44488
0.7081	0.07319	0.07335	0.58757	0.59128
0.9441	0.06069	0.06030	0.70895	0.71182
1.1802	0.04762	0.04693	0.80419	0.80578

$\sigma = 4.2366$		$_r\beta_1 = 0$		$_r\beta_2 = 2.897$
Standard Score Deviation $\frac{X}{\sigma}$	Frequency		Cumulative frequency	
	Exact	Normal	Exact	Normal
1.4162	0.03531	0.03454	0.87481	0.87500
1.6523	0.02469	0.02405	0.92419	0.92330
1.8883	0.01625	0.01584	0.95669	0.95518
2.1243	0.01003	0.00986	0.97675	0.97506
2.3604	0.00579	0.00581	0.98833	0.98780
2.5964	0.00311	0.00324	0.99455	0.99335
2.8325	0.00155	0.00171	0.99765	0.99682
3.0685	0.00071	0.00085	0.99907	0.99855
3.3045	0.00030	0.00040	0.99967	0.99938
3.5406	0.00011	0.00018	0.99989	0.99974
3.7766	0.00004	0.00007	0.99997	0.99990
4.0126	0.00001	0.00003	0.99999	0.99996

Table VIII. Comparison between the Normal Integral and a 20-fold non-replacement sample drawn from Universe F.

$\sigma = 3.017$		$_r\beta_1 = 0$		$_r\beta_2 = 2.83$
Standard Score Deviation $\frac{X}{\sigma}$	Frequency		Cumulative frequency	
	Exact	Normal	Exact	Normal
0	0.12940	0.13223	0.12940	0.13168
0.3314	0.12303	0.12516	0.37546	0.38094
0.6629	0.10567	0.10614	0.58680	0.59269
0.9944	0.08175	0.08065	0.75030	0.75398
1.3258	0.05671	0.05490	0.86372	0.86417
1.6573	0.03501	0.03349	0.93374	0.93168
1.9887	0.01903	0.01830	0.97180	0.96879
2.3202	0.00897	0.00896	0.98974	0.98707
2.6516	0.00359	0.00393	0.99692	0.99514
2.9831	0.00118	0.00154	0.99928	0.99835
3.3145	0.00030	0.00055	0.99988	0.99951
3.6460	0.00005	0.00017	0.99998	0.99986
3.9775	0.00001	0.00005	1.00000	0.99997

Summary.

Moments of the score-sum and mean-score distribution for the general case of sampling without replacement from a universe of N objects assignable to n classes each specifiable by a definitive score disclose three outstanding common properties: (a) symmetry of the

half-universe sample distribution: (b) platykurtic *r*-fold sampling distributions even for a highly leptokurtic u-s-d, for sampling fractions in the range $F \simeq 0.22$ to $F \sim 0.79$; (c) close correspondence between the Pearson coefficients of a *one-third* sample from a sizeable ($N \geqslant 40$) universe and their normal values.

Numerical examinations of distributions referable to different sampling fractions from universes defined by widely different contours illustrate these conclusions: and moments of difference distributions generated by sampling without replacement are also derived.

Résumé.

La somme des moments et la distribution des moments moyens d'un échantillon obtenu par tirages exhaustifs d'une population de N individus répartis en n classes, délimitables par des caractéristiques définies, démontre trois qualités marquées en commun:

a) Distribution symétrique dans un échantillon composé de la moitié de la population.

b) Distributions platycurtics dans un échantillon doublé r fois, aussi pour un u-s-d fort leptocurtic, pour les fractions de l'échantillon dans la sphère de $F \simeq 0.22$ à $F \simeq 0.79$.

c) Un accord satisfaisant entre les coefficients de *Pearson* dans la distribution d'un échantillon composé d'un tiers d'une population d'une certaine grandeur ($N > 40$) et leur valeurs normales.

Des études numériques de distributions es rapportant aux fractions de populations définies par des limites très divergentes éclaircissent ces conclusions. Des moments de distributions de différence obtenues par tirages exhaustifs sont aussi dérivés.

Zusammenfassung.

Die Momentsumme und die Verteilung der Mittelmomente in einer allgemeinen Stichprobe, welche ohne Rückstellung einer Population von N Individuen, verteilt auf n Klassen, entnommen wurde, eine jede durch bestimmte Charakteristika spezifiziert, weisen drei ausgeprägte, gemeinsame Eigenschaften auf:

a) Symmetrie in einer Stichprobeverteilung, bestehend aus der halben Population;

b) flachkegelförmige, r-faltige Stichprobenverteilungen, selbst für stark leptokurtisches u-s-d, für Stichprobenfraktionen innerhalb des Gebietes $F \simeq 0.22$ bis $F \simeq 0.79$;

c) nahe Übereinstimmung zwischen den *Pearson*-Koeffizienten

in einer Stichprobe, bestehend aus einem Drittel einer einigermaßen großen Population ($N > 40$), und deren Normalwerten.

Ziffermäßige Untersuchungen von Verteilungen, welche zu Fraktionen von Populationen, bestimmt durch voneinander weit verschiedene Grenzen, gerechnet werden können, erläutern diese Schlußsätze; Momente in Differenzverteilungen, welche man durch Stichprobenentnahme ohne Rückstellung erhalten hat, werden ebenfalls abgeleitet.

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ERFAHRUNGEN MIT DER ESSEN-MÖLLERSCHEN FORMEL BEI DER ERBBIOLOGISCHEN VATERSCHAFTSBEGUTACHTUNG

Von DIETRICH WICHMANN, Tübingen

Die zunehmende Bedeutung der erbbiologischen Begutachtungen bei Paternitätsklagen ließ bei den Richtern und bei den Sachverständigen den Wunsch aufkommen, das Ergebnis der erbbiologischen Untersuchung in einen zahlenmäßigen Ausdruck zu fassen. Die von *Essen-Möller* entwickelte Formel, die übrigens nicht die einzige Möglichkeit darstellt, ist wohl am bekanntesten geworden. Die Ansichten über ihren Wert sind allerdings in der Fachwelt geteilt. Während *E. Fischer*, *F. Lenz*, *O. Reche* und besonders *W. Ludwig* und *M. Weninger* sich ablehnend verhalten, stimmten – wenn auch teilweise mit Vorbehalten – *E. Geyer*, *L. Löffler*, *P. Kramp*, *S. Koller* und *K. Tuppa* ihr zu. Es ist daher angezeigt, die *praktische* Überprüfung ihrer Eignung an einer weiteren Untersuchungsserie durchzuführen, nachdem *E. Geyer* mit positivem Ergebnis vorangegangen war.

Zu Beginn sei aber auf die wesentlichsten Einwände eingegangen, wobei gemäß unserem Thema die theoretischen Vorbehalte in den Hintergrund treten sollen. So wird einer biologischen Denkweise

eine mechanisch-rechnerische gegenübergestellt; es gibt aber nur eine, die allen Wissenschaften gemeinsam ist, nämlich die logische. Auch wenn man nicht der Ansicht ist, daß die Mathematik die Königin der Wissenschaften ist, wird man zugeben müssen, daß sie die Denkgesetze in die kürzeste und allgemeinste Form bringt.

Weiterhin hat man eingewandt, daß für ein Ereignis, das schon eingetreten ist, keine objektive Wahrscheinlichkeit festgestellt werden kann. Als Ereignis eingetreten ist in unserem Falle aber nur die *Erzeugung* eines Kindes, nicht dagegen die *Vaterschaft* eines bestimmten Präsumtivvaters. Denn ob ein angegebener Mann der Vater eines Kindes ist oder nicht, ist ja gerade das Problem, vor dem der Gutachter steht und das er mit erbbiologischen Methoden lösen soll. Richtig ist jedoch, daß das Wahrscheinlichkeitsurteil die Sicherheit der Feststellung betrifft. Es ist zuzugeben, daß der Ausdruck „Vaterschaftswahrscheinlichkeit" unkorrekt ist. Tatsächlich wird mit Hilfe der *Essen-Möller*schen Formel die Wahrscheinlichkeit bestimmt, mit der ein in Anspruch genommener Mann der Gruppe der wahren oder der der falschen Väter zugeordnet werden kann. Mit anderen Worten: die Begutachtung mit der *Essen-Möller*schen Formel behandelt also den *Einzelfall als Stichprobe aus einem größeren Vergleichsmaterial mit bestimmten Grenzfestsetzungen.*

Essen-Möller geht von folgenden Gedanken aus:

1. Der Vater wird ein Merkmal seines Kindes häufiger besitzen als ein falsch angegebener Mann.

2. Die Übereinstimmung in einem seltenen Merkmal ist von größerer Bedeutung als die in einem häufigen Merkmal.

3. Bei gleichzeitiger Mutter-Kind-Ähnlichkeit ist das Auftreten des kindlichen Merkmals beim Präsumptivvater von geringerer Beweiskraft als wenn es der Mutter fehlen würde.

4. Die Sicherheit des Schlusses auf eine Vaterschaft wächst mit der Zahl der Übereinstimmungen.

Die Richtigkeit dieser Grundsätze ist unbestreitbar, sie dürften wohl von allen Gutachtern angewandt werden, wobei das Schwergewicht je nach Erfahrung auf den einen oder andern Satz gelegt wird. *Neu* ist bei *Essen-Möller* die Kombination dieser Sätze in *einer* Formel.

Der Hinweis auf mögliche *Manifestationsschwankungen* eines Gens ist von schwerwiegender Bedeutung. Bekanntlich ist *Geyer* in seiner ersten Arbeit von bestimmten Genhäufigkeiten und Erb-

gängen ausgegangen, deren Stichhaltigkeit teilweise durchaus bestreitbar ist. Daher möchte auch *Löffler* die Anwendung der Formel vorerst nur auf die serologischen Merkmale beschränken. Es ist aber durchaus nicht notwendig, in die Formel mit bestimmten Erbgängen einzugehen, wie die Kritiker anscheinend annehmen. Der sogenannte „kritische Wert", d. h. die Merkmalshäufigkeit bei falschen Vätern (Y) im Verhältnis zu der bei wahren Vätern (X), läßt sich auch *empirisch* durch Auswerten von Bevölkerungs- und Familienuntersuchungen gewinnen. Allerdings sollten diese Untersuchungsserien nicht zu klein sein, vor allem wenn man eine weitgehende Klassenunterteilung zu Grunde legt. Diese Schwäche hat übrigens auch *Geyer* selbst bemerkt und in einer weiteren, anscheinend ziemlich unbekannt gebliebenen Arbeit die empirische Berechnung des „kritischen Wertes" an Hand der Thenarbemusterung gezeigt.

Durch Verwendung eines großen Familienmaterials gehen aber auch mögliche Manifestationsschwankungen bzw. Klassifikationsschwierigkeiten bei Merkmalen mit kontinuierlichen Übergängen in die Formel ein. Hierdurch werden „Ausschlüsse" unmöglich, die Seltenheit derartiger Konstellationen wird jedoch mit genügendem Gewicht in der Gesamtbeurteilung bewertet. Die Verwendung empirischen Familienmaterials bietet aber noch weitere Vorteile. Da die Häufigkeiten bestimmter Kind-Mutter-Vater-Konstellationen ausgezählt werden, braucht man eine etwa vorhandene *Paarungssiebung* bei der Partnerwahl nicht besonders zu berücksichtigen. Sorgt man weiterhin dafür, daß die Filialgeneration aus jugendlichen Probanden besteht, so kann man auf zusätzliche Berechnungen verzichten, die den *Altersunterschied* ausgleichen sollen. Wenn in der Filialgeneration Jungen und Mädchen zu gleichen Teilen vorhanden sind, brauchen *Geschlechtsunterschiede* in der Merkmalsverteilung nicht in einer besonderen Berechnung ausgewertet zu werden. Wenn das Familienmaterial groß genug ist, können aber selbstverständlich Jungen-Mutter-Vater- und Mädchen-Mutter-Vater-Gruppen gebildet werden, wodurch die Aussage etwas schärfer wird. Sind Alters- und Geschlechtsunterschiede nachweisbar, dann sollte man den Wert Y „Merkmalshäufigkeit in der Bevölkerung" nicht aus der *Gesamt*bevölkerung sondern aus der *erwachsenen männlichen* Bevölkerung berechnen.

Die unterschiedlichen Häufigkeiten in verschiedenen Bevölkerungen sind selbstverständlich nicht ohne Einfluß auf das Ergebnis. Es heißt aber ihr Gewicht überschätzen, wenn man für jede in Betracht kommende Gegend *gesondert* die Merkmalshäufigkeiten fest-

stellen müßte, so wünschenswert die anthropologisch-erbbiologische Bestandesaufnahme einzelner Bevölkerungen auch ist. Ein Beispiel mag dies zeigen. Die Häufigkeit des Blutgruppengens B beträgt in Westdeutschland (Bonn) 6,8 %, in Ostdeutschland (Königsberg) ist es hingegen doppelt so häufig: 13,2 %. Die Konstellation Kind B, Mutter 0, Präsumtivvater B ist also in Westdeutschland ohne Zweifel beweiskräftiger als in Ostdeutschland. Im vorstehenden Fall läßt sich für Bonn eine Zuordnungswahrscheinlichkeit von 88,5 %, für Königsberg eine solche von 82,0 % berechnen. Für den gesamtdeutschen Durchschnitt (Genhäufigkeit 9 %) beträgt sie 85,5 %. Es müssen also schon *sehr erhebliche* Unterschiede zwischen den Bevölkerungen vorliegen, wenn sie das Ergebnis *entscheidend* beeinflussen sollen. Erst wenn das dominante Gen rund 3mal so häufig ist, z. B. Gen A im Reichsdurchschnitt 29 %, ist ein wesentliches Absinken der Zuordnungswahrscheinlichkeit zu beobachten, sie beträgt dann bei der Konstellation Kind A, Mutter 0, Präsumptivvater A 67,5 %.

Schließlich sind noch Bedenken dagegen geäußert worden, verschiedenartige Merkmale in einer Formel zu vereinigen. Der Einwand kann aber nicht als stichhaltig gelten, da derartige Merkmale auch bei der abschließenden Gesamtbeurteilung eines Falles verwertet werden. Im Gegenteil, gerade die rechnerische Behandlung einer erbbiologischen Untersuchung *bewertet* erst die unterschiedlichen Aussagen der einzelnen Merkmale unter Berücksichtigung ihrer Häufigkeit in der Bevölkerung und in bestimmten Familienkombinationen. Auch in der *Essen-Möller*schen Formel kann ein einziges Merkmal eine größere Wahrscheinlichkeit oder Unwahrscheinlichkeit begründen als 10 andere. Eine mehr gefühlsmäßige Abschätzung bringt unerwünschte subjektive Elemente in die Beurteilung und kann daher zu mehr oder minder großen Fehlschlüssen führen.

Hier ist vielleicht der richtige Ort, kurz den Ausdruck „Additionsbeweis" zu besprechen, der der juristischen Begriffswelt entstammt und der öfters in erbbiologischen Gutachten auftritt. Hierunter wird die Häufung von einzelnen Indizien verstanden, die addiert zu einem Gesamtbeweis zusammengefaßt werden. Dieser Begriff ist aber falsch, da in einem Gesamturteil die einzelnen Wahrscheinlichkeiten nicht *additiv* sondern *multiplikativ* mit einander verknüpft sind, was auch in der *Essen-Möller*schen Formel geschieht.

In den Naturwissenschaften ist es üblich, die Richtigkeit oder Unrichtigkeit einer theoretischen Überlegung im *Experiment* zu

überprüfen. Hierfür dienten uns als Material 120 Vaterschaftsgutachten des Anthropologischen Instituts der Universität Tübingen, die von *Gieseler* und drei Mitarbeitern erstattet wurden. Zu beurteilen waren insgesamt 216 Männer, von denen 112 mit mehr oder minder großer Wahrscheinlichkeit als Väter bezeichnet werden konnten, während bei 104 Männern die Vaterschaft abgelehnt wurde.

Das notwendige Vergleichsmaterial boten Bevölkerungsuntersuchungen, die zum großen Teil unter *Gieselers* Leitung von dessen Schülern und Mitarbeitern erhoben und teilweise auch veröffentlicht wurden. Weiterhin wurden Häufigkeitsziffern auch aus dem gesamten Gutachtenmaterial des Instituts berechnet. Die notwendigen Familien konnten zum Teil aus den Bevölkerungsuntersuchungen, zum Teil aus dem Gutachtenmaterial zusammengestellt werden. Außerdem wurde speziell für diesen Zweck gesammeltes Familienmaterial herangezogen. Die Zahl der Elternpaare schwankte zwischen 300–500, die der dazugehörigen Kinder zwischen 500–1000. Bestimmte Erbgänge wurden nur bei den Blutgruppen des AB0-Systems und den Blutkörperchenmerkmalen des MN-Systems zugrunde gelegt, die Genhäufigkeiten wurden aus den Werten *W. Fischers* für Gesamtdeutschland berechnet. Für alle anderen verwendeten Merkmale – insgesamt 31 – wurden die kritischen Werte empirisch berechnet. Fräulein Dr. *Ehrhardt* und Herrn Dr. *Heinrich* habe ich für die freundliche Überlassung einiger von ihnen berechneter Bewertungstabellen zu danken.

Für die Berechnung wurden 3 Maße, 10 morphologische Merkmale des Kopfes und des Gesichtes, 2 serologische Merkmale und 16 Merkmale des Hautleistensystems an Fingerbeeren, Handflächen und Fußsohlen benutzt. Die Auswahl dieser Merkmale soll durchaus nicht als endgültig betrachtet werden, es war aber ein Kompromiß zu schließen zwischen den Merkmalen, die bei der erbbiologischen Untersuchung erhoben worden waren einerseits und den an und für sich vorhandenen Unterlagen andererseits. Eine Erweiterung und ein teilweiser Austausch sind zukünftig vorgesehen.

Das Ergebnis dieser Auswertung wird aus der Abbildung 1 ersichtlich.

Der Mittelwert der Zuordnungswahrscheinlichkeit beträgt für die 112 wahren Väter 92,48 $\pm$ 0,75 %, für die 104 zu Unrecht in Anspruch genommenen Männer hingegen nur 26,34 $\pm$ 1,82 %. Es liegt also ein einwandfreier Unterschied (66,14 $\pm$ 1,97 %) vor. Weiterhin ist der Unterschied in der Anordnung der Varianten be-

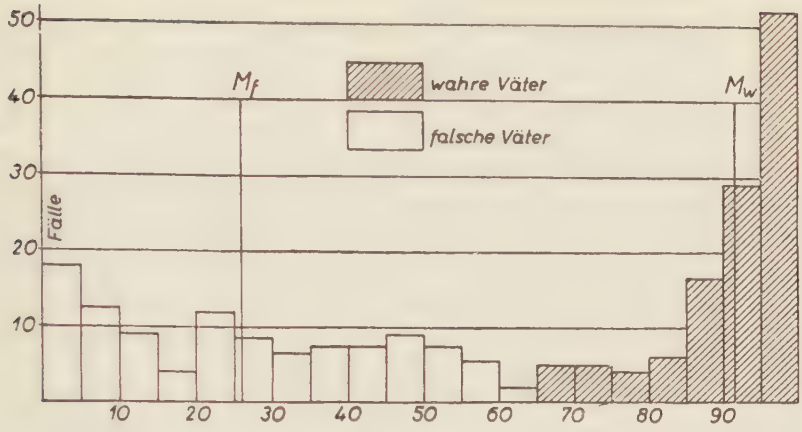

Abb. 1. Zuordnungswahrscheinlichkeit in %.

merkenswert, beide Gruppen zeigen zwar eine schiefe Verteilung, diese ist jedoch bei den falschen Vätern wesentlich weitläufiger ($\sigma = 17{,}94 \pm 1{,}25$) als bei den wahren Vätern ($\sigma = 7{,}72 \pm 0{,}52$). Für keinen der wahren Väter ergab sich ein Wert, der im Bereich der falschen Väter lag und umgekehrt, jedoch ist in der Mitte der Tafel eine Annäherung feststellbar, der niedrigste Wert beträgt bei den wahren Vätern 68 %, bei den falschen der höchste 64 %. Bei noch größeren Serien ist also mit Überschneidungen zu rechnen. Derartige Grenzfälle sind aber nicht durch die *Essen-Möller*sche Formel bedingt, sie sind auch bei mehr gefühlsmäßiger Abschätzung des Untersuchungsergebnisses durchaus möglich.

Ist in Fällen mit 2 Präsumtivvätern der eine dem Kinde ähnlich der andere hingegen unähnlich, so läßt sich sagen, daß der ähnliche Mann der Erzeuger des Kindes ist, während der unähnliche Mann praktisch ausgeschieden werden kann. In einer weiteren Arbeit hat *Essen-Möller* zusammen mit dem Mathematiker *Quensel* seine bisherige Formel in dieser Richtung fortentwickelt. Es kann nicht meine Aufgabe sein, diese Formel im einzelnen vorzuführen und abzuleiten, die Originalarbeit muß unbedingt eingesehen werden. Es sei hier nur soviel gesagt, daß der jeweilige „kritische Wert" des anderen Mannes der Summe aus beiden gegenüber gestellt wird. Hat z. B. der Mann A eine Wahrscheinlichkeit von 70 %, der Mann B eine solche von 20 %, so ergeben sich folgende Beziehungen, wenn man voraussetzt, daß sich der tatsächliche Vater unter den Untersuchten befindet:

$$\text{Für den Mann A } \frac{4{,}00}{4{,}43} = 90{,}3\ \%,$$

$$\text{für den Mann B } \frac{0{,}43}{4{,}43} = 9{,}7\ \%.$$

War bisher die Wahrscheinlichkeit für A $3\,{}^1/_2$mal größer als die für B, so wird sie nach dem neuen Verfahren mehr als 9mal so groß. Zu beachten ist außerdem, daß die Wahrscheinlichkeiten für 2 Männer sich immer zu 100 ergänzen, wodurch man eine zusätzliche Rechenkontrolle erhält. Hätte sich hingegen für den Mann B gesondert auch eine Wahrscheinlichkeit von 70 % ergeben, so läßt sich für beide Männer eine Wahrscheinlichkeit von $\frac{0{,}43}{0{,}86} = 50\ \%$ errechnen, d. h. der Fall ist nicht zu entscheiden, es muß auch mit der Vaterschaft eines bisher unbekannten Mannes gerechnet werden.

Diese erweiterte Berechnung wurde für die 89 Zwei-Mann-Fälle unseres Materials durchgeführt. Als wichtigste Ergebnisse sind festzustellen: Ohne Gegenüberstellung hatte $^1/_7$ der wahren Väter eine geringere Wahrscheinlichkeit als 85 %, $^1/_5$ überschritt die 99 %-Grenze. Bei Gegenüberstellung haben sämtliche wahren Väter eine Wahrscheinlichkeit von 85 % und darüber, $^9/_{10}$ überschreiten die 90 %-Grenze statt bisher $^3/_4$, die Hälfte aller wahren Väter liegt über 99 %, während ohne Gegenüberstellung nur $^1/_5$ darüber lag.

Noch klarer ist der Fortschritt in der Zuordnungswahrscheinlichkeit der falsch angegebenen Männer erkennbar. Ohne Gegenüberstellung belief sich bei $^2/_5$ die Wahrscheinlichkeit auf 15 % und darunter, nur ein knappes Zehntel hatte eine geringere Zuordnungswahrscheinlichkeit als 1 %. Bei der Gegenüberstellung verschieben sich die Verhältnisse entsprechend wie bei den wahren Vätern, kein falsch angegebener Mann besitzt eine höhere Wahrscheinlichkeit als 15 %, $^9/_{10}$ haben eine geringere Wahrscheinlichkeit als 10 % (bisher $^1/_3$), die Hälfte aller falschen Väter hat eine Wahrscheinlichkeit von 1 % und darunter.

	Wahre Väter Zuordnungswahrscheinlichkeit in %							
	65–69	70–74	75–79	80–84	85–89	90–94	95–x	zusammen
Ohne Gegenüberstellung	2	4	2	4	10	18	49	89
Bei Gegenüberstellung	—	—	—	—	5	11	73	89

	Falsche Väter Zuordnungswahrscheinlichkeit in %							
	x–5	6–10	11–20	21–30	31–40	41–50	51–x	zusammen
Ohne Gegenüberstellung	17	10	11	17	11	11	12	89
Bei Gegenüberstellung	73	11	5	—	—	—	—	89

Es ist vielleicht von Interesse, die 3 aus der Abbildung ersichtlichen Grenzfälle hier bei der Gegenüberstellung vorzuführen. Für einen Zeugen, der „mit an Sicherheit grenzender Wahrscheinlichkeit" als Vater bezeichnet worden war, ließ sich nur eine Wahrscheinlichkeit von 68,5 % errechnen, für den Beklagten betrug die Wahrscheinlichkeit nur 6 %. Bei Gegenüberstellung ergaben sich 97,1 % für den Zeugen und 2,9 % für den Beklagten. In einem anderen Fall konnte für den Beklagten, der mit „größter Wahrscheinlichkeit" als Vater bezeichnet worden war, nur eine Wahrscheinlichkeit von 68,4 % errechnet werden, für den Mehrverkehrszeugen 21,8 %. Bei einem direkten Vergleich belief sich aber die Wahrscheinlichkeit für den Zeugen nur noch auf 10,6 %, während sie beim Beklagten auf 89,4 % anstieg. Im dritten Fall lautete die Wahrscheinlichkeit für den „sehr unwahrscheinlichen" Zeugen 64,0 %, für den „sehr wahrscheinlichen" Beklagten 92,1 %. Bei der rechnerischen Gegenüberstellung sank die Wahrscheinlichkeit für den Zeugen auf 13,2 %, während sie für den Beklagten mit 86,8 % nahezu das 6fache des Zeugen betrug.

Zusammenfassung.

Abschließend kommen wir daher zu folgenden Ergebnissen:

1. Nach unserer Erfahrung erscheint die *Essen-Möller*sche Methode – insbesondere in ihrer erweiterten Form – für die erbbiologische Begutachtung als durchaus geeignet.

2. Für die rechnerische Auswertung sollten möglichst klar definierbare Merkmale herangezogen werden. Die serologischen Merkmale können in diesem Zusammenhang eine große Rolle spielen.

3. Es ist angebracht, für die Berechnung der „kritischen Werte" empirisch gewonnenes Vergleichsmaterial zu Grunde zu legen, nur bei serologischen Merkmalen kann man mit bestimmten Genhäufigkeiten in die Rechnung eingehen.

4. Nach dem derzeitigen Stande der Dinge kann die *Essen-Möller*sche Formel im Rahmen der Gesamtuntersuchung dem Gutachter wertvolle Hinweise bieten. Ihre alleinige Verwendung bei der Begutachtung in Paternitätsklagen halten wir noch für verfrüht, sie ist aber als Ziel anzustreben.

Summary.

The following results were obtained:

1. According to my experience the method of *Essen-Möller* appears well adapted for a calculation of the probability of disputed paternity.

2. For the numerical evaluation one should use characteristics as clearly definable as possible. Serological characters may be of great importance in this connection.

3. It is advisable to base the calculation of the "critical values" on an empirically obtained control material. Only for serological characters is it possible to count upon definite gene frequences.

4. Within the scope of the examination as a whole the formula of *Essen-Möller* may give the investigator valuable indications for the present. It seems however premature to use only this formula for a statement regarding a case of disputed paternity, though it is a goal to strive for.

Résumé.

Nous sommes arrivés aux résultats suivants:

1. Selon notre opinion il semble qu'on puisse faire usage de la formule d'*Essen-Möller*, particulièrement dans sa forme élargie, pour un avis héréditaire-biologique.

2. Pour l'évaluation numérique il faut employer des caractères aussi nettement définissables que possible. Dans ce contexte les caractères biologiques du sang peuvent jouer un grand rôle.

3. En calculant les «valeurs critiques» il convient de prendre pour base des matériaux de comparaison obtenus empiriquement. Seulement quand il s'agit des caractères biologiques du sang on peut s'attendre aux fréquences de gènes définies.

4. Appliquée dans les limites de l'examen dans son ensemble la formule d'*Essen-Möller* peut donner à l'examinateur des indications de valeur. Il semble pourtant qu'on ne puisse encore employer que cette formule en donnant son avis sur des affaires de paternité, bien que cela soit un but à poursuivre.

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CAUSES OF EXCESS MALE MORTALITY IN MAN

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Summary.

The study of the relevant data and the discussion of the various hypotheses which have been put forward for the explanation of excess male mortality, leads to the conclusion that the explanations of the phenomenon, according to which its cause is sought in increased occupational risk, or in genetical differences between the sexes, or in sex-dimorphic physiological and endocrinological differences, are not mutually exclusive, but that each of them has its proper sphere of application for the explanation of the phenomenon in question at different ages of life, and for different aspects of it.

I.

The notion of the impartiality of death which gave rise to the concept of death as The Leveller and inspired *Holbein* to the creation of his "Dance of Death" woodcuts, with Death approaching indiscriminately high and low, The Duke and The Pedlar, is changing fast.

We know now,—if we did not in a vague way know before,—that death is not so impartial as it seemed to our less sophisticated and less informed ancestors. Although he will eventually grab The Duke as he does The Pedlar, yet he hesitates somewhat when confronting the former, and is rather quick in eliminating the latter. He seems to have less scruples in slaying the lower classes. He thus, in a way, differentiates between rich and poor, high and low. Moreover, he appears to have his preferences with regard to the sexes, and that rather differently from what might be expected.

In the eyes of death the male appears to be the weaker sex. We have arrived today at the conclusion that, for some reason or other, the male, in virtue of his maleness, is less viable than the female. Under certain circumstances the male, because of a greater inherent

fragility, succumbs more easily to the force of death. This phenomenon has been reviewed by *F. A. E. Crew*, F.R.S., not only for the human species but also for other mammals, birds and insects, both in the open and under controlled conditions of experimentation, (in his presidential address for Section D [Zoology] of the British Association in September, 1937). He has shown that the whole course of sex mortality in pre-natal life, infancy and subsequent age periods, is consistent with the view that the male in man is the inherently weaker sex, more prone to death from diseases at all times[1]. It may be that during life the male is still the stronger, but when it comes to dying he has decidedly less resistance. We may also express this by saying that the force of death is stronger when directed against the male. For a comprehensive presentation and thorough discussion of the problems involved, see *C. Gini*, «Il sesso dal punto di vista statistico», Metron, Roma.

Looking closer we find that this effect is due to many of the weapons death uses,—the causes of death,—having themselves a greater affinity to the male. It becomes more and more clear that it is only the lack of detailed knowledge which was conducive to the formation of the striking overall picture of death as the leveller, whose action appeared as the very embodiment of chance. As our knowledge grows, this picture in which we distinguished at first nothing but life and death, light and dark, begins to show all the complexity of the network of correlations between the causes of death and the classes of humanity.

II.

The phenomenon of male excess mortality, though possessing a fairly high degree of generality as regards country, time and age of life, is by no means uniform in all these respects, nor can it be said to have complete generality. It varies to some extent from country to country, and changes from time to time as regards intensity, and for certain ages of life it gives place to the opposite phenomenon, female excess mortality. The following tables of the ratio: male over female death rate which are based upon material by *P. Delaporte*[2] show that there are characteristic differences in excess male mortality from place to place and from time to time.

[1] "The Sex Ratio". Presidential address for section D (Zoology) of the British Association, September 1937.

[2] *P. Delaporte.* Evolution de la mortalité en Europe depuis l'origine des statistiques de l'état civil, «Statistique Generale de la France», Paris 1941.

Table 1. Sweden.

Age	1750	1780	1800	1820	1840	1860	1880	1900	1920
0	1,11	1,11	1,14	1,14	1,17	1,18	1,19	1,25	1,30
1	1,20	1,03	1,05	1,06	1,02	1,08	1,06	1,08	1,77
2	**0,98**	1,00	1,03	1,03	1,11	1,11	1,09	1,08	1,22
3	**0,97**	1,03	1,13	1,10	1,05	1,08	1,03	1,06	1,11
4	1,15	1,18	1,05	1,07	1,05	1,02	1,06	1,00	1,22
5	1,11	1,12	1,04	1,11	1,17	1,10	1,06	1,00	1,18
6	1,08	1,08	1,03	1,06	1,14	1,13	1,06	1,02	1,16
7	1,08	1,08	1,01	1,13	1,07	1,11	1,06	**0,98**	1,17
8	1,11	1,07	1,06	1,14	1,12	1,12	1,02	1,00	1,17
9	1,06	1,02	1,09	1,28	1,22	1,05	1,06	1,00	1,17
10	1,11	1,04	1,11	1,32	1,15	1,02	1,11	1,11	1,11
11	1,10	1,10	1,05	1,23	1,14	1,09	**0,98**	1,04	1,11
12	1,01	1,11	1,12	1,16	1,17	1,02	**0,98**	**0,93**	1,00
13	1,03	1,10	1,14	1,11	1,14	1,03	**0,95**	**0,83**	**0,86**
14	1,05	1,10	1,14	1,09	1,17	**0,98**	**0,95**	**0,76**	**0,77**
15	1,08	1,13	1,14	1,09	1,14	**0,98**	**0,95**	**0,75**	1,00
16	1,09	1,15	1,13	1,06	1,11	**0,98**	**0,96**	**0,97**	
17	1,11	1,15	1,15	1,06	1,08	1,04	1,00	1,15	
18	1,15	1,16	1,18	1,10	1,08	1,11	1,06	1,21	
19	1,18	1,15	1,22	1,11	1,13	1,18	1,13	1,20	
20	1,19	1,16	1,25	1,18	1,21	1,22	1,15	1,15	
21	1,20	1,16	1,26	1,25	1,26	1,17	1,11	1,18	
22	1,19	1,17	1,25	1,26	1,27	1,15	1,11	1,13	
23	1,20	1,16	1,24	1,25	1,24	1,10	1,09	1,09	
24	1,21	1,16	1,21	1,26	1,22	1,08	1,07	1,07	
25	1,19	1,17	1,21	1,22	1,19	1,08	1,05	1,05	
26	1,15	1,16	1,19	1,23	1,21	1,08	1,07	1,02	
27	1,11	1,13	1,17	1,22	1,17	1,06	1,07	1,05	
28	1,08	1,11	1,18	1,21	1,17	1,06	1,07	1,02	
29	1,06	1,10	1,17	1,19	1,15	1,05	1,05	1,02	
30	1,07	1,09	1,14	1,17	1,14	1,05	1,05	1,00	
31	1,08	1,08	1,14	1,15	1,13	1,03	1,04	1,01	
32	1,08	1,10	1,14	1,15	1,11	1,04	1,02	**0,98**	
33	1,09	1,09	1,12	1,13	1,11	1,03	1,02	**0,98**	
34	1,10	1,11	1,13	1,13	1,11	1,03	1,02	1,00	
35	1,13	1,13	1,15	1,11	1,10	1,04	1,02	1,05	
36	1,15	1,15	1,17	1,13	1,10	1,04	1,00		
37	1,18	1,18	1,18	1,13	1,10	1,04	1,00		
38	1,18	1,19	1,22	1,14	1,10	1,06	1,00		
39	1,18	1,22	1,24	1,16	1,11	1,05	1,00		
40	1,21	1,23	1,25	1,17	1,12	1,07	1,03		
41	1,22	1,25	1,27	1,19	1,14	1,08	1,03		
42	1,24	1,27	1,30	1,21	1,15	1,09	1,03		
43	1,26	1,29	1,31	1,22	1,17	1,10	1,05		
44	1,28	1,31	1,32	1,24	1,20	1,13	1,06		
45	1,29	1,32	1,34	1,25	1,22	1,15	1,08		
46	1,29	1,34	1,32	1,27	1,24	1,16	1,09		
47	1,31	1,35	1,35	1,29	1,26	1,17	1,10		
48	1,31	1,35	1,35	1,29	1,26	1,16	1,10		
49	1,34	1,36	1,37	1,32	1,29	1,17	1,09		
50	1,35	1,36	1,36	1,32	1,32	1,19	1,10		
51	1,36	1,37	1,35	1,33	1,33	1,20	1,07		
52	1,35	1,37	1,35	1,33	1,31	1,20	1,09		

Tables 1 to 7 after *N. Federici*, "Statistica", Vol. X, pp. 274–320 (1950).

Age	1750	1780	1800	1820	1840	1860	1880	1900	1920
53	1,31	1,37	1,35	1,33	1,32	1,20	1,07		
54	1,26	1,33	1,35	1,31	1,31	1,20	1,09		
55	1,26	1,28	1,32	1,29	1,29	1,20	1,09		
56	1,27	1,24	1,30	1,29	1,26	1,20			
57	1,29	1,25	1,25	1,28	1,24	1,17			
58	1,29	1,24	1,21	1,28	1,22	1,18			
59	1,26	1,23	1,19	1,28	1,19	1,19			
60	1,22	1,20	1,19	1,25	1,22	1,16			
61	1,18	1,18	1,19	1,22	1,23	1,16			
62	1,15	1,16	1,17	1,19	1,23	1,17			
63	1,16	1,14	1,15	1,17	1,20	1,17			
64	1,20	1,14	1,14	1,20	1,16	1,14			
65	1,20	1,19	1,12	1,19	1,14	1,11			
66	1,17	1,23	1,11	1,17	1,16	1,10			
67	1,15	1,19	1,12	1,15	1,15	1,11			
68	1,13	1,18	1,17	1,13	1,13	1,08			
69	1,12	1,15	1,18	1,12	1,11	1,07			
70	1,10	1,13	1,17	1,13	1,09	1,05			
71	1,09	1,12	1,15	1,18	1,07	1,04			
72	1,10	1,11	1,14	1,16	1,14	1,04			
73	1,12	1,10	1,13	1,13	1,11	1,04			
74	1,11	1,11	1,12	1,12	1,09	1,02			
75	1,10	1,13	1,11	1,11	1,09	1,01			
76	1,09	1,11	1,11	1,09	1,06				
77	1,08	1,12	1,15	1,08	1,06				
78	1,11	1,10	1,13	1,10	1,04				
79	1,11	1,10	1,12	1,11	1,07				
80	1,09	1,13	1,12	1,10	1,06				
81	1,05	1,11	1,12	1,09	1,04				
82	1,03	1,08	1,13	1,12	1,03				
83	1,02	1,07	1,13	1,10	1,09				
84	1,02	1,07	1,12	1,09	1,04				
85	1,01	1,09	1,13	1,09	1,03				
86	1,02	1,10	1,14	1,08	1,04				
87	1,09	1,15	1,13	1,09	1,03				
88	1,11	1,20	1,19	1,10	1,03				
89	1,12	1,18	1,15	1,13	**0,99**				
90	1,12	1,15	1,18	1,18	1,01				
91	1,13	1,15	1,16	1,14	1,00				
92	1,11	1,14	1,17	1,12	**0,99**				
93	1,13	1,13	1,16	1,11	1,00				
94	1,13	1,14	1,17	1,13	1,01				
95	1,12	1,15	1,17	1,13	1,03				
96	1,12	1,14	1,18	1,09					
97	1,14	1,14	1,18	1,13					
98	1,13	1,15	1,19	1,14					
99	1,11	1,14	1,18	1,14					
100	1,12	1,15	1,17	1,14					

The distribution of excess male mortality over the ages of life in the various countries of Europe differs considerably as regards extent and degree, and furthermore, the phenomenon has changed differently in different countries with time.

From the record we have for *Sweden*, which goes back to 1750, it

Table 2. Norway.

Age	1840	1860	1880	1900	1920	Age	1840	1860	1880
0		1,18	1,19	1,20	1,21	46	1,09	1,07	1,14
1		1,03	1,03	1,10	**0,92**	47	1,11	1,06	1,17
2		1,04	1,02	1,20	1,00	48	1,12	1,08	1,21
3		1,00	**0,88**	1,17	1,33	49	1,16	1,10	1,24
4		1,00	**0,92**	1,00	1,00	50	1,17	1,13	1,29
5		1,05	1,05	1,04	1,00	51	1,20	1,15	
6		**0,97**	**0,99**	1,10	1,00	52	1,22	1,16	
7	1,00	**0,99**	**0,98**	1,08	1,00	53	1,23	1,19	
8	1,00	**0,99**	1,06	1,06	1,00	54	1,25	1,21	
9	1,03	**0,98**	**0,98**	1,00	1,00	55	1,26	1,22	
10	1,00	**0,98**	**0,98**	**0,97**	1,00	56	1,27	1,23	
11	1,00	**0,98**	**0,95**	**0,94**		57	1,28	1,25	
12	1,00	**0,95**	**0,93**	**0,84**		58	1,29	1,26	
13	1,02	**0,93**	**0,89**	**0,78**		59	1,25	1,28	
14	1,00	**0,91**	**0,88**	**0,83**		60	1,24	1,27	
15	1,00	**0,96**	1,00	**0,87**		61	1,22	1,24	
16	1,00	1,06	1,15	1,00		62	1,21	1,24	
17	1,02	1,17	1,33	1,20		63	1,19	1,24	
18	1,16	1,35	1,46	1,35		64	1,16	1,21	
19	1,34	1,46	1,53	1,54		65	1,15	1,18	
20	1,47	1,51	1,57	1,55		66	1,17	1,17	
21	1,58	1,56	1,62	1,57		67	1,21	1,20	
22	1,58	1,57	1,63	1,57		68	1,19	1,23	
23	1,51	1,53	1,59	1,51		69	1,16	1,26	
24	1,43	1,45	1,55	1,45		70	1,14	1,29	
25	1,35	1,40	1,49	1,42		71	1,18		
26	1,26	1,33	1,43	1,41		72	1,15		
27	1,19	1,25	1,36	1,38		73	1,19		
28	1,13	1,21	1,32	1,31		74	1,16		
29	1,08	1,15	1,25	1,21		75	1,14		
30	1,04	1,12	1,19	1,22		76	1,13		
31	1,03	1,06	1,16			77	1,12		
32	1,03	1,03	1,13			78	1,12		
33	1,00	1,00	1,09			79	1,12		
34	1,00	**0,99**	1,06			80	1,15		
35	1,01	**0,99**	1,03			81	1,14		
36	1,00	**0,99**	1,03			82	1,12		
37	1,00	**0,99**	1,01			83	1,10		
38	1,01	**0,99**	1,01			84	1,09		
39	1,02	1,00	1,03			85	1,09		
40	1,01	1,01	1,03			86	1,07		
41	1,02	1,01	1,04			87	1,05		
42	1,03	1,04	1,06			88	1,04		
43	1,02	1,03	1,07			89	1,08		
44	1,03	1,05	1,10			90	1,10		
45	1,06	1,06	1,11						

is clear that the excess male mortality occurred, at least as far back as 1750. There is further in all countries a general rise in excess male mortality as time proceeds, although there are exemptions to this trend. One of these exemptions is *Sweden* where the excess male mortality has declined in the last 100 years or so.

Table 3. England.

Age	1840	1860	1880	1900	1920	Age	1840	1860	1880
0	1,27	1,26	1,24	1,27	1,28	51	1,27	1,24	1,29
1	1,06	1,04	1,05	1,08	1,10	52	1,26	1,24	1,32
2	1,00	1,00	**0,95**	1,06	1,06	53	1,26	1,24	1,34
3	**0,99**	1,00	1,00	1,00	1,00	54	1,27	1,23	1,37
4	1,00	1,10	1,02	**0,94**	1,13	55	1,27	1,23	1,37
5	1,06	1,04	1,00	**0,96**	1,20	56	1,25	1,25	1,36
6	1,05	1,04	1,00	1,02	1,15	57	1,23	1,27	1,36
7	**0,99**	1,07	1,11	1,06	1,14	58	1,23	1,31	1,36
8	**0,96**	1,03	1,06	1,00	1,11	59	1,26	1,31	
9	**0,90**	1,02	1,04	1,00	1 13	60	1,30	1,30	
10	**0,90**	1,00	1,00	1,00	1,08	61	1,31	1,28	
11	**0,89**	**0,92**	1,00	1,00	1,00	62	1,30	1,29	
12	**0,83**	**0,94**	1,00	1,00	1,00	63	1,27	1,31	
13	**0,84**	**0,97**	1,00	**0,95**	1,00	64	1,26	1,33	
14	**0,80**	**0,97**	**0,96**	1,00	**0,94**	65	1,25	1,33	
15	**0,88**	**0,98**	**0,93**	1,00	**0,95**	66	1,26	1,30	
16	**0,89**	**0,95**	1,00	1,00	1,00	67	1,28	1,27	
17	**0,92**	**0,96**	1,03	1,04	1,04	68	1,30	1,23	
18	**0,96**	**0,96**	1,06	1,03	1,08	69	1,28	1,30	
19	**0,99**	**0,98**	1,05	1,03		70	1,24	1,35	
20	1,01	**9,98**	1,08	1,06		71	1,21	1,36	
21	1,01	**0,98**	1,10	1,06		72	1,18	1,33	
22	1,02	1,00	1,10	1,06		73	1,17	1,30	
23	1,04	1,02	1,10	1,06		74	1,15	1,27	
24	1,05	1,02	1,12	1,06		75	1,14	1,26	
25	1,05	1,05	1,12	1,09		76	1,18	1,25	
26	1,05	1,04	1,11	1,09		77	1,20	1,26	
27	1,03	1,06	1,11	1,12		78	1,20	1,30	
28	1,04	1,06	1,13	1,09		79	1,18		
29	1,03	1,06	1,15	1,09		80	1,17		
30	1,04	1,05	1,15	1,09		81	1,17		
30	1,04	1,05	1,15	1,09		81	1,17		
31	1,04	1,07	1,17	1,12		82	1,18		
32	1,05	1,08	1,19	1,12		83	1,17		
33	1,06	1,09	1,21	1,12		84	1,16		
34	1,08	1,10	1,22	1,12		85	1,15		
35	1,11	1,11	1,24	1,12		86	1,15		
36	1,12	1,12	1,24	1,12		87	1,12		
37	1,13	1,13	1,25	1,15		88	1,11		
38	1,13	1,15	1,26	1,15		89	1,12		
39	1,15	1,17	1,25			90	1,11		
40	1,17	1,17	1,27			91	1,11		
41	1,18	1,19	1,28			92	1,11		
42	1,19	1,21	1,29			93	1,11		
43	1,20	1,21	1,32			94	1,10		
44	1,20	1,23	1,30			95	1,09		
45	1,22	1,24	1,30			96	1,08		
46	1,23	1,25	1,32			97	1,06		
47	1,23	1,25	1,32			98	1,06		
48	1,26	1,25	1,31			99	1,05		
49	1,26	1,25	1,31			100	1,04		
50	1,25	1,26	1,30						

Looking at the tables, one finds that there are considerable differences from country to country as regards the distribution of excess male mortality over the ages of life. Most countries show an excess female mortality for certain age groups. Thus in *Norway*,

Table 4. France.

1820	1840	1860	1880	1900	1920	Age	1820	1840	1860	1880
1,15	1,17	1,00	1,19	1,21	1,24	50	1,12	1,26	1,40	1,56
1,03	1,23	1,10	1,04	**0,95**	1,30	51	1,14	1,26	1,40	1,59
1,02	1,10	1,06	1,07	1,03	1,02	52	1,16	1,27	1,37	1,61
1,01	1,03	**0,98**	1,02	1,10	1,04	53	1,18	1,27	1,40	1,62
0,98	1,02	1,01	1,11	**0,97**	1,03	54	1,20	1,31	1,43	1,64
1,01	1,01	1,02	**0,97**	**0,93**	1,00	55	1,19	1,30	1,43	1,61
0,98	1,00	**0,93**	**0,95**	**0,91**	1,00	56	1,19	1,31	1,42	1,59
1,02	**0,91**	**0,92**	**0,94**	**0,92**	1,05	57	1,19	1,30	1,44	1,60
1,03	**0,95**	**0,89**	**0,92**	**0,91**	1,00	58	1,17	1,30	1,44	1,59
0,98	**0,85**	**0,79**	**0,93**	**0,93**	1,00	59	1,17	1,30	1,45	1,63
0,86	**0,80**	**0,81**	**0,91**	**0,92**	1,00	60	1,17	1,30	1,44	1,63
0,80	**0,78**	**0,79**	**0,88**	**0,96**	**0,94**	61	1,17	1,31	1,45	1,62
0,77	**0,77**	**0,73**	**0,85**	**0,88**	**0,94**	62	1,18	1,31	1,45	1,63
0,76	**0,79**	**0,76**	**0,87**	**0,85**	**0,89**	63	1,17	1,30	1,45	1,62
0,77	**0,81**	**0,82**	**0,86**	**0,75**	**0,91**	64	1,16	1,29	1,44	1,61
0,80	**0,85**	**0,79**	**0,87**	**0,76**	**0,93**	65	1,16	1,27	1,41	1,61
0,82	**0,87**	**0,79**	**0,88**	**0,90**	**0,91**	66	1,15	1,26	1,41	1,62
0,84	**0,88**	**0,85**	**0,92**	**0,93**		67	1,15	1,26	1,40	1,59
0,88	**0,92**	**0,91**	**0,99**	1,04		68	1,17	1,27	1,43	1,58
0,92	**0,98**	1,03	1,05	1,12		69	1,16	1,26	1,39	1,57
0,99	1,02	1,14	1,14	1,19		70	1,14	1,24	1,36	1,52
1,14	1,12	1,18	1,11	1,19		71	1,14	1,24	1,35	1,51
1,23	1,28	1,41	1,19	1,17		72	1,16	1,25	1,36	1,50
1,24	1,36	1,37	1,18	1,15		73	1,16	1,26	1,36	1,49
1,20	1,26	1,32	1,14	1,09		74	1,14	1,25	1,38	1,49
1,10	1,15	1,22	1,10	1,07		75	1,15	1,24	1,36	1,52
0,99	1,00	1,13	1,07	1,07		76	1,18	1,25	1,35	1,49
0,94	**0,91**	1,07	1,06	1,09		77	1,17	1,26	1,35	1,47
0,90	**0,90**	1,01	1,07	1,14		78	1,15	1,22	1,33	1,45
0,87	**0,87**	**0,98**	1,09	1,18		79	1,15	1,21	1,30	1,41
0,86	**0,89**	**0,99**	1,13	1,25		80	1,15	1,20	1,27	1,37
0,87	**0,91**	1,00	1,15	1,30		81	1,15	1,20	1,25	1,34
0,88	**0,94**	1,01	1,18	1,35		82	1,13	1,18	1,25	1,32
0,90	**0,96**	1,05	1,20	1,39		83	1,13	1,17	1,22	1,29
0,91	**0,97**	1,08	1,22	1,43		84	1,11	1,15	1,19	1,25
0,93	**0,99**	1,10	1,25	1,48		85	1,11	1,13	1,16	1,21
0,93	1,01	1,13	1,26	1,52		86	1,13	1,13	1,14	1,19
0,95	1,03	1,17	1,28			87	1,13	1,10	1,11	1,16
0,96	1,07	1,19	1,29			88	1,12	1,13	1,13	1,13
0,98	1,09	1,23	1,30			89	1,11	1,11	1,11	1,11
1,01	1,12	1,25	1,32			90	1,10	1,10	1,10	1,10
1,03	1,14	1,29	1,34			91	1,09	1,09	1,09	1,09
1,07	1,14	1,31	1,37			92	1,07	1,07	1,07	1,07
1,09	1,17	1,34	1,39			93	1,07	1,07	1,07	1,07
1,10	1,21	1,37	1,43			94	1,06	1,06	1,06	1,06
1,11	1,23	1,39	1,46			95	1,06	1,06	1,06	1,06
1,12	1,24	1,41	1,47			96	1,06	1,06	1,06	1,06
1,12	1,24	1,42	1,48			97	1,05	1,05	1,05	1,05
1,11	1,24	1,43	1,50			98	1,04	1,04	1,04	1,04
1,11	1,25	1,42	1,52			99	1,05	1,05	1,05	1,05

Denmark, we find a persistent excess female mortality for the age groups between 10 and 15, and 30 and 35 approximately. In the *Netherlands* the second age group is very much extended and reaches from approximately 26 to 43. There is also a third age group for

Table 5. Denmark.

Age	1840	1860	1880	1900	1920	Age	1840	1860	1880
0				1,21	1,07	51	1,37	1,27	1,04
1				1,05	1,15	52	1,41	1,28	1,05
2				**0,98**	1,20	53	1,45	1,28	1,06
3				1,00	1,00	54	1,46	1,27	1,06
4				1,00	1,00	55	1,44	1,27	1,05
5				**0,98**	1,05	56	1,42	1,27	
6				1,00	1,13	57	1,43	1,26	
7			**0,99**	**0,90**	1,15	58	1,41	1,26	
8			**0,93**	**0,96**	1,18	59	1,39	1,22	
9			**0,89**	1,00	1,22	60	1,38	1,22	
10		**0,97**	**0,85**	**0,89**	1,25	61	1,35	1,22	
11	**0,89**	**0,92**	**0,78**	**0,89**	1,29	62	1,30	1,20	
12	**0,85**	**0,83**	**0,76**	**0,89**	1,13	63	1,31	1,17	
13	**0,80**	**0,80**	**0 74**	**0,89**	1,25	64	1,31	1,14	
14	**0,76**	**0,75**	**0,78**	**0,85**	1,30	65	1,31	1,16	
15	**0,76**	**0,73**	**0,82**	**0,83**	1,25	66	1,29	1,19	
16	**0,78**	**0,75**	**0,86**	**0,88**		67	1,22	1,12	
17	**0,82**	**0,80**	**0,93**	**0,93**		68	1,17	1,09	
18	**0,87**	**0,87**	1,00	1,00		69	1,12	1,05	
19	**0,97**	**0,94**	1,07	1,03		70	1,15	1,03	
20	1,06	1,02	1,11	1,03		71	1,15	1,03	
21	1,17	1,09	1,11	1,06		72	1,12	1,00	
22	1,19	1,12	1,11	1,03		73	1,11	1,01	
23	1,16	1,09	1,09	1,03		74	1,08	1,01	
24	1,07	1,04	1,00	**0,94**		75	1,07	1,00	
25	1,00	1,00	1,00	**0,91**		76	1,06		
26	**0,93**	**0,94**	**0,98**	**0,88**		77	1,05		
27	**0,91**	**0,90**	**0,94**	**0,82**		78	1,07		
28	**0,88**	**0,86**	**0,94**	**0,82**		79	1,05		
29	**0,86**	**0,84**	**9,94**	**0,82**		80	1,02		
30	**0,86**	**0,86**	**0,92**	**0,80**		81	1,00		
31	**0,86**	**0,86**	**0,94**	**0,80**		82	1,00		
32	**0,85**	**0,87**	**0,92**	**0,83**		83	1,04		
33	**0,86**	**0,90**	**0,94**	**0,83**		84	1,06		
34	**0,87**	**0,93**	**0,82**	**0,86**		85	1,07		
35	**0,89**	**0,96**	**0,92**	**0,86**		86	1,10		
36	**0,91**	**0,99**	**0,94**			87	1,06		
37	**0,93**	**1,01**	**0,94**			88	1,04		
38	**0,95**	1,04	**0,94**			89	1,05		
39	**0,97**	1,07	**0,96**			90	1,08		
40	**0,99**	1,10	**0,96**			91	1,09		
41	1,01	1,11	**0,98**			92	1,10		
42	1,02	1,13	**0,98**			93	1,11		
43	1,06	1,16	1,02			94	1,10		
44	1,09	1,17	1,02			95	1,11		
45	1,13	1,18	1,02			96	1,10		
46	1,17	1,20	1,01			97	1,11		
47	1,21	1,22	1,03			98	1,10		
48	1,25	1,22	1,03			99	1,11		
49	1,30	1,24	1,03			100	1,11		
50	1,32	1,26	1,05						

Table 6. Netherlands.

Age	1840	1860	1880	1900	1920
0	1,17	1,22	1,21	1,20	1,31
1	1,01	1,01	1,07	1,08	1,11
2	1,00	**0,99**	1,11	1,25	1,14
3	1,00	1,23	1,19	1,44	1,25
4	**0,95**	1,08	1,25	1,19	1,17
5	1,06	1,02	1,09	1,08	1,00
6	1,06	**0,98**	1,11	1,05	1,16
7	1,02	1,00	1,09	1,13	1,13
8	1,06	**0,99**	1,04	1,12	1,15
9	1,00	1,02	1,05	1,18	1,17
10	**0,97**	1,02	1,03	1,15	1,20
11	**0,98**	**0,96**	**0,94**	1,05	1,22
12	1,02	**0,95**	**0,84**	**0,90**	1,22
13	**0,88**	**0,89**	**0,76**	1,00	1,10
14	**0,84**	**0,83**	**0,82**	1,05	1,10
15	**0,83**	**0,79**	**0,89**	1,04	1,10
16	**0,86**	**0,86**	**0,93**	1,12	
17	**0,94**	1,00	1,02	1,15	
18	1,04	1,19	1,16	1,29	
19	1,16	1,23	1,25	1,38	
20	1,22	1,27	1,28	1,13	
21	1,26	1,28	1,28	1,07	
22	1,23	1,25	1,19	1,03	
23	1,20	1,18	1,10	1,00	
24	1,14	1,13	1,04	**0,97**	
25	1,07	1,07	**0,98**	**0,93**	
26	1,02	1,01	**0,96**	**0,96**	
27	**0,96**	0,96	**0,92**	**0,96**	
28	**0,91**	0,92	**0,91**	**0,96**	
29	**0,87**	0,90	**0,89**	**0,96**	
30	**0,86**	0,86	**0,87**	**0,93**	
31	**0,84**	0,85	**0,87**	**0,89**	
32	**0,83**	0,85	**0,84**	**0,93**	
33	**0,81**	0,85	**0,82**	**0,93**	
34	**0,81**	0,85	**0,82**		
35	**0,81**	0,86	**0,81**		
36	**0,82**	0,87	**0,81**		
37	**0,83**	0,87	**0,79**		
38	**0,84**	0,89	**0,79**		
39	**0,85**	0,89	**0,80**		
40	**0,88**	0,92	**0,81**		
41	**0,90**	0,94	**0,80**		
42	**0,93**	0,97	**0,82**		
43	**0,97**	0,99	**0,84**		
44	1,01	1,01	**0,89**		
45	1,06	1,04	**0,90**		
46	1,09	1,08	**0,89**		
47	1,12	1,11	**0,90**		
48	1,15	1,14	**0,96**		
49	1,18	1,16	**0,99**		
50	1,21	1,17	**0,95**		
51	1,24	1,18	**0,96**		
52	1,26	1,19	**0,96**		
53	1,27	1,17	**0,95**		
54	1,28	1,18	**0,95**		
55	1,29	1,17	**0,94**		
56	1,26	1,18	**0,92**		
57	1,26	1,18			
58	1,26	1,15			
59	1,25	1,12			
60	1,21	1,15			
61	1,19	1,14			
62	1,20	1,15			
63	1,22	1,12			
64	1,22	1,09			
65	1,18	1,14			
66	1,15	1,11			
67	1,13	1,08			
68	1,11	1,07			
69	1,15	1,05			
70	1,15	1,04			
71	1,13	1,07			
72	1,11	1,06			
73	1,09	1,06			
74	1,09	1,04			
75	1,08	1,06			
76	1,07	1,07			
77	1,10				
78	1,10				
79	1,07				
80	1,06				
81	1,06				
82	1,05				
83	1,06				
84	1,07				
85	1,09				
86	1,09				
87	1,07				
88	1,05				
89	1,06				
90	1,07				
91	1,08				
92	1,09				
93	1,08				
94	1,07				
95	1,08				
96	1,09				
97	1,08				
98	1,08				
99	1,09				
100	1,08				

Table 7.

	Germany				Finland		
Age	1880	1900	1920	Age	1880	1900	1920
0			1,20	0	1,19	1,22	1,21
1			1,07	1	1,05	**0,61**	1,05
2			1,07	2	1,03	1,02	1,10
3			1,00	3	1,03	**0,95**	1,13
4			1,00	4	1,00	**0,96**	1,06
5	1,00	**0,99**	1,02	5	**0,93**	**0,91**	1,00
6	1,06	**0,95**	1,07	6	**0,92**	**0,91**	1,00
7	**0,90**	1,03	1,05	7	**0,93**	**0,89**	1,05
8	**0,95**	**0,98**	1,06	8	**0,91**	**0,90**	1,06
9	**0,97**	**0,95**	1,07	9	**0,89**	**0,96**	1,07
10	**0,91**	**0,89**	1,00	10	**0,95**	**0,95**	1,08
11	**0,84**	**0,88**	**0,90**	11	**0,89**	**0,90**	1,09
12	**0,77**	**0,82**	**0,80**	12	**0,88**	**0,90**	1,20
13	**0,73**	**0,78**	**0,72**	13	**0,87**	**0,95**	1,30
14	**0,70**	**0,74**	**0,68**	14	**0,82**	1,05	1,18
15	**0,76**	**0,79**	**0,81**	15	**0,91**	1,13	
16	**0,83**	**0,87**		16	1,00	1,16	
17	**0,90**	**0,98**		17	1,08	1,22	
18	1,00	1,06		18	1,15	1,32	
19	1,10	1,13		19	1,19	1,33	
20	1,11	1,50		20	1,18	1,32	
21	1,11	1,43		21	1,17	1,31	
22	1,11	1,40		22	1,15	1,30	
23	1,09	1,35		23	1,08	1,26	
24	1,07	1,32		24	1,06	1,26	
25	1,06	1,27		25	1,04	1,20	
26	1,06	1,26		26	1,02	1,17	
27	1,04	1,21		27	1,00	1,08	
28	1,01	1,18		28	**0,98**	1,03	
29	1,00	1,20		29	**0,98**	1,00	
30	**0,99**	1,20		30	**0,96**	1,00	
31	**0,97**	1,21		31	**0,95**	**0,97**	
32	**0,97**	1,25		32	**0,96**	1,03	
33	**0,97**	1,29		33	**0,96**	1,12	
34	**0,99**	1,33		34	**0,96**	1,23	
35	1,00	1,37		35	**0,98**		
36	1,03			36	**0,98**		
37	1,04			37	1,02		
38	1,06			38	1,03		
39	1,09			39	1,07		
40	1,12			40	1,08		
41	1,17			41	1,11		
42	1,20			42	1,13		
43	1,25			43	1,14		
44	1,30			44	1,17		
45	1,32			45	1,18		
46	1,37			46	1,17		
47	1,41			47	1,18		
48	1,46			48	1,19		
49	1,51			49	1,21		
50	1,54			50	1,22		
51	1,63			51	1,23		
52	1,72			52	1,24		
53	1,80						

Table 7.

	Italy				Switzerland		
Age	1880	1900	1920	Age	1880	1900	1920
0	1,10	1,10	1,11	0	1,18	1,20	1,93
1	1,00	**0,97**	1,02	1	1,00	1,04	1,15
2	**0,98**	1,00	1,05	2	**0,99**	1,02	1,00
3	**0,97**	**0,95**	1,09	3	1,03	1,04	1,15
4	**0,91**	0,93	1,00	4	1,02	1,02	1,06
5	1,02	**0,93**	1,02	5	1,00	1,00	1,00
6	1,00	**0,92**	**0,97**	6	1,00	**0,97**	**0,96**
7	**0,94**	**0,92**	1,00	7	1,02	**0,94**	1,05
8	**0,90**	**0,90**	1,00	8	**0,98**	**0,96**	1,06
9	**0,88**	**0,83**	1,04	9	**0,95**	**0,96**	1,00
10	**0,84**	**0,79**	1,05	10	**0,91**	**0,96**	1,00
11	**0,85**	**0,80**	1,05	11	**0,90**	**0,91**	1,00
12	**0,87**	**0,86**	1,06	12	**0,84**	**0,83**	**0,92**
13	**0,87**	**0,81**		13	**0,81**	**0,80**	
14	**0,85**	**0,81**		14	**0,75**	**0,79**	
15	**0,81**	**0,85**		15	**0,74**	**0,84**	
16	**0,82**	**0,88**		16	**0,81**	**0,91**	
17	**0,85**	**0,91**		17	**0,86**	**0,94**	
18	**0,89**	1,00		18	**0,92**	**0,92**	
19	**0,92**	1,06		19	**0,94**	**0,95**	
20	**0,97**	1,12		20	**0,96**	**0,95**	
21	1,01	1,18		21	**0,96**	**0,95**	
22	1,01	1,20		22	**0,96**	**0,98**	
23	1,00	1,20		23	**0,97**	1,00	
24	**0,99**	1,18		24	**0,98**	1,00	
25	**0,94**	1,10		25	**0,98**	1,03	
26	**0,92**	1,00		26	1,00	1,05	
27	**0,89**	**0,98**		27	1,00	1,05	
28	**0,88**	**0,96**		28	1,00	1,08	
29	**0,87**	**0,98**		29	1,02	1,13	
30	**0,87**	1,02		30	1,03	1,16	
31	**0,89**	1,07		31	1,03	1,23	
32	**0,90**	1,14		32	1,07	1,29	
33	**0,91**			33	1,08		
34	**0,91**			34	1,10		
35	**0,93**			35	1,13		
36	**0,94**			37	1,18		
37	**0,94**			36	1,15		
38	**0,96**			38	1,21		
39	**0,99**			39	1,24		
40	1,00			40	1,27		
41	1,03			41	1,31		
42	1,06			42	1,33		
43	1,08			43	1,37		
44	1,11			44	1,41		
45	1,14			45	1,44		
46	1,18			46	1,46		
47	1,20			47	1,48		
48	1,23			48	1,49		
49	1,26			49	1,55		
50	1,27			50	1,58		
51	1,27			51	1,59		
52	1,26			52	1,59		

female excess mortality in the *Netherlands* between 45 and 55, at least in the year 1880.

In *England*, we used to have a pronounced female excess mortality between 10 and 20 which, however in this century has contracted to the age group 14–15. In *France* we find a similar contraction of excess female mortality from 10–20 to between 11 and 16. In *France*, however, there used to be a large excess female mortality between 36 and 39 which, with time, has disappeared. The same contraction phenomenon of the two age groups for excess female mortality mentioned above is also found in *Finland*, *Germany* and *Italy; Switzerland* shows only the first of these groups and that contracts very much in this century.

The explanation of this phenomenon seems fairly obvious. The first period represents that of adolescence and puberty and it is well-known that the crisis of puberty in girls is accompanied by deeper organic changes than in boys. The second period for which there is, in general, an excess female mortality is that in which the female sex is exposed to the hazards of child-bearing. It is quite in accordance with this interpretation that, as these hazards become less and less, through advances in medicine, that the age group with less and less, through advances in medicine, the age group with excess female mortality should contract more and more as time goes on, and that in some countries with a high standard of living and of hygiene it should give place by and by to excess male mortality for these very ages. This has happened, for instance in *England*, *Sweden*, *Norway* and *Switzerland*.

III.

The most obvious explanation of excess male mortality is an increased risk for males in the environmental conditions and an increased occupational risk. For example, excess mortality in males due to violence is, no doubt, very often due to the fact that males are exposed, or do expose themselves, to a greater extent, to risks of that description. Since, however, during the present century conditions of work have improved considerably as regards safety measures and hygiene, and hours of work have been reduced, one would expept a decrease of excess male mortality on that account, whereas the opposite is the case: excess male mortality is on the increase in our time. Another fact which may be adduced against this type of explanation is that industrial occupation of females is on the increase, and that therefore they are more exposed to occupational risk than before.

Finally there is a pronounced excess male mortality in the youngest age groups and also in the highest age groups for which the explanation of occupation or environmental risk is obviously not suitable.

The hypothesis of the increased occupational and environmental risk run by males as an explanation for excess male mortality can therefore have only a limited validity. It may explain certain special cases of the phenomenon and it may have application to those ages which expose themselves to risks of dying by acts of violence, and to occupations where predominantly male workers are exposed to the pathogenic influence of mineral or metal dust, gas, fumes, etc., but it cannot be regarded as a sufficient explanation of the phenomenon in general.

IV.

The phenomenon in question attracted first attention in the form of excess male infant mortality. It was investigated by *F. Lenz*[1], for *Germany* as a whole, *Bavaria*, *France*, *Spain*, *Italy*, *Austria*, *Hungary*, *England*, *Sweden* and *Norway*. The hypothesis put forward by him was that certain genetic differences between the sexes may be regarded as responsible for excess male mortality.

This hypothesis is based upon the fact that the genetic structure of males, in the human species, is principally different from that of females.

In man, the male is the heterogametic sex, and as such possesses one x-chromosome, whereas the female, as the homogametic sex, possesses two. Thus, the female has two parallel sets of genes whereas the male has, strictly speaking, no such parallel set, because the differential segments of his y-chromosome do not exactly correspond to those of his x-chromosome.

It follows that if a recessive gene for a certain disease or condition is carried in a differential segment of the x-chromosome, it is at once uncovered in the case of the heterogametic individual, and if in its action such a gene is disadvantageous, deleterious or lethal, it will find expression in the phenotype of the individual. If, on the other hand, the individual is homogametic there is always a chance that the same differential segment which carries the recessive gene in one chromosome may carry in the other chromosome a compensating

[1] Die Übersterblichkeit der Knaben im Lichte der Erblichkeitslehre. Archiv für Hygiene, XCIII. pp. 126–150.

gene and the expression of the recessive gene in the phenotype would be prevented.

Lenz found corroboration for his hypothesis in the fact that increase of infant excess male mortality with time was, in general, accompanied by a decrease of the death rate of infants, males and females together. He argues as follows: if the lack of resistance to certain diseases were due to certain recessive genes, and if these recessive characteristics were sex-linked or at least sex-limited, then these diseases would find expression more often as general health conditions improved. As a consequence, the downward trend in the general mortality rate of infants should be accompanied by an upward trend of excess male mortality.

This he found confirmed in all the countries for which he had collected data. His method was to compare the ratio of male over female infant death rate with the general infant death rate and calculate the correlation co-efficient for the two series. He obtained, invariably, a significant negative correlation which he regarded as support for his hypothesis that excess male mortality was due to innate differences between the sexes. To make his argument clear, let us suppose that in a given population males had a death rate of 1 % and females of 0.8 %, so that out of 100 000 males we would expect 1000 to die per year, and out of 100 000 females 800, the excess male death rate would then be 1.25. If now general health conditions deteriorate, say, due to an epidemic and the death rate of each sex increases by, say, 5 %, the male death rate would go up to 1.5 % and the female death rate to 1.3 %, and the excess male death rate would now be only 1.1 %.

Conversely, an improvement in health conditions affecting both sexes in the same way would result in an increase in the excess male mortality.

The certainly remarkable phenomenon of the negative correlation between the two series, is not confined to infant mortality but can be extended to mortality for all ages taken together, and thus to the general death rate.

If we wish to compare the overall death rate for persons (male and female together) with the excess male mortality, for a series of years, we must apply two corrections: one for the difference in age composition of the population from year to year, and another to correct for difference in age composition between males and females in a given year. We therefore use in place of the crude death rates

for male and female together, the comparative mortality index (the year 1938 is taken as the base year), and in place of the crude ratio of male to female mortality, the adjusted ratio (table 8).

The two series of data are for quinquennial intervals from 1941–1945 (table 3 of the Registrar General's Statistical Review of England and Wales for the year 1949, tables Part I Medical, H. M. Stationery Office, 1951).

Table 8.

Year	Comparative Mortality Indices (Base year 1938 taken as unity)	Adjusted Ratio of Male to Female Mortality
1841–1845	2,179	1,096
1846–1850	2,360	1,088
1851–1855	2,276	1,099
1856–1860	2,177	1,095
1861–1865	2,253	1,116
1866–1870	2,242	1,132
1871–1875	2,220	1,150
1876–1880	2,130	1,160
1881–1885	2,018	1,152
1886–1890	1,997	1,164
1891–1895	1,993	1,161
1896–1900	1,885	1,178
1901–1905	1,716	1,191
1906–1910	1,572	1,198
1911–1915	1,495	1,226
191o–1920	1,450	1,282
1921–1925	1,220	1,240
1926–1930	1,167	1,268
1931–1935	1,104	1,276
1936–1940	1,070	1,337
1941–1945	0,956	1,414

The correlation co-efficient results as 0.97, which is highly significant and in full agreement with the theory put forward by *Lenz.*

However, in regarding highly significant negative correlation co-efficients between infant or overall mortality and excess male mortality as a support for the genetical hypothesis, it should be borne in mind that we are dealing with the correlation of two time series, and that although two events may appear correlated in time they may have nothing to do with one another. It is therefore only in conjunction with the reasonableness of the genetical hypothesis that we can regard the negative correlation co-efficient between the two series as a support for the hypothesis.

The hypothesis has been the subject of a thorough investigation by *M. Greenwood* and *E. M. Newbold*[1].

For the correct evaluation of the hypothesis it is necessary to consider the results reached by *Greenwood* and *Newbold*. They examined infant mortality in England and Wales taking into consideration both the static and the dynamic variation, that is, the variation according to place and to time. For all causes together they found *Lenz*'s criterion, that is the negative correlation between the mortality of infants and excess mortality of infant males, confirmed for both dynamic and static variation. For special causes of death they found it confirmed for diarrhoea, and, on the whole, also for tuberculosis, congenital debility and respiratory diseases. For diphtheria, croup and measles they found a negative correlation for certain series, but not for all. For whooping cough, for which female mortality is notoriously in excess, they found the correlation positive for all groups. They did not regard their results as conclusive evidence for the correctness of *Lenz*'s theory.

They also investigated the theoretical side of correlating the two series and it is of interest to consider their analysis. What *Lenz* correlated was the ratio of male to female death rate and the death rate of infants, male and female together.

If x is the rate of male mortality that is in this case the ratio of deaths under one year of age to births in the year and y that of female mortality, what *Lenz* is correlating is $x\,y$ and $\frac{Ax + By}{A + B}$.[2]

But in his series $\frac{A}{A + B}$ and $\frac{B}{A + B}$ are approximately constant, and neglecting the difference in the sex ratio at birth, each may be taken as equal to 0.5, so that approximately we have the correlation of $\frac{x}{y}$ with $x + y$. To a first approximation this is equal to

$$\frac{\frac{\sigma_x^2}{x} - \frac{\sigma_y^2}{y} + r_{xy}\sigma_x\sigma_y\left(\frac{1}{x} - \frac{1}{y}\right)}{\sqrt{\frac{\sigma_x^2}{x^2} + \frac{\sigma_y^2}{y^2} - \frac{2r_{xy}\sigma_x\sigma_y}{xy}}\ \sqrt{\sigma_x^2 + \sigma_y^2 + 2r_{xy}\sigma_x\sigma_y}} \quad \ldots \quad (1)$$

[1] *M. Greenwood* and *E. M. Newbold* on the Excess Mortality of Males in the first year of life. Biometrica, Vol. 17, 1925, p. 327.

[2] A, B are the numbers of male and female births, resp. in the year.

The last term of the numerator becomes negative if $\bar{x}$ is greater than $\bar{y}$, that is, if the average male mortality exceeds the average female mortality. It follows that if the first term $\frac{\sigma_x^2}{\bar{x}} - \frac{\sigma_y^2}{\bar{y}}$ is not positive and greater in absolute value than the last term, then the correlation co-efficient between $\frac{x}{y}$ and $x + y$ must be negative, therefore, if $\frac{\sigma_y^2}{\bar{y}} > \frac{\sigma_x^2}{\bar{x}}$ and if $\bar{x} > \bar{y}$ then the correlation co-efficient will be negative.

It appears now from *Greenwood*'s paper that the two conditions under which the correlation co-efficient becomes negative are again correlated in the case of simple sampling, that is if the variation within the series of death rates can be regarded as following the *Bernoullian* Law.

In that case, the standard deviation of the percentage, $\sqrt{\frac{pq}{n}}$, is related to the mean value of the p's, and the absolute variation increases with the value of the percentage. The increase, however, is such that it increases more slowly than the mean value, with the consequence that as the percentage increases, the relative variation, that is the standard deviation divided by the mean, decreases. We may therefore say that for the case of simple sampling an excess of $\frac{\bar{x}}{\bar{y}}$ is accompanied by an excess of the relative variation of y over that of x.

It appears now that *Greenwood* regarded these algebraic consequences of a difference between $\bar{x}$ and $\bar{y}$ and of the corresponding relative variations as something which detracted from the value of the hypothesis as an explanation of the phenomenon in question. It goes without saying that such an interpretation is not justified. The negative correlation co-efficient, provided it is significant, is not less real because we understand its mathematical structure and because it appears as an algebraical necessity in the case of simple sampling.

In addition to this, *Greenwood* himself has shown that the condition of simple sampling hardly ever applies to these experimental series. It follows, therefore, that the negative correlations as far

as they result from his series cannot be regarded as algebraical consequences of the mean values of male and female mortality. He found that even in the case of non-simple sampling a high absolute variance and a small relative variance did result for that sex which has the higher average mortality. Instead of regarding this as a confirmation of the reality of the negative correlation co-efficient, he considered it, in a way, as an argument against *Lenz*'s theory.

It is only for the cases where the mortality of either sex is very small that, although the average male mortality may be higher than the average female mortality, and the absolute variation of the former also higher than that of the latter, the coefficient of variation, or the relative variability, does not follow the usual pattern, that is, decreases as the absolute variation increases. This accounts for the correlation in such cases being sometimes positive. This is the case for instance for some series of the epidemic diseases like diphtheria, croup and measles. Apart from the fact that for these diseases the causes are environmental, and one would not therefore expect, according to *Lenz*'s theory, an excess male mortality due to recessive characteristics to express itself, it appears that the variation in these cases comes very near to that of the simple sampling scheme, which would account for the fact that there is only very little and hardly significant correlation. That in the case of whooping cough, the correlation co-efficient results as positive, is again quite in keeping with the theory, because the excess mortality here is simply reversed and according to the structure of the formula, there must result in this case a positive correlation co-efficient.

Greenwood's analysis, therefore, need by no means be considered as invalidating the genetic theory, nor, to do him justice, does he make any statement to that effect. He only considers his results not completely satisfactory for upholding the *Lenz*'s theory without, however, attempting to come to a definite conclusion as to inherent differences in male and female variation.

VI.

We have next to consider the phenomenon of the change of excess male mortality with age of life, and the changes which the excess male mortality for specific age groups has undergone with time.

Regarding the trend of excess male mortality with age of life, there appears to be a systematic difference for excess mortality

according to the age of the persons concerned. In the first year of life the excess mortality is very high; for the extreme ages 70 and higher it is fairly low, it reaches a peak between about 20 and 60. This appears to be a more or less general phenomenon at all times and in all countries although there are marked differences in this respect. For England and Wales, the excess male mortality for specified age groups expressed as a percentage of the female death rate is given in table 9 (based upon table 5, Registrar General's Statistical Review of England and Wales for the year 1949. Tables Pt. I Medical).

In order to test the differences in excess male mortality between the ages of life one might adopt various methods. In any test it would be a question of ascertaining whether the differences can be regarded as real or whether they ought to be still considered only fluctuations due to chance.

The method adopted by the Italian writer *Nora Federici*[1] is that of the calculus of co-graduation. The index of co-graduation can be calculated either between territorial series or between the temporal series. If there was no selection as regards age of life, that is if for successive years of life the excess male mortality remained more or less the same, then the co-efficient of graduation should be more or less equal to unity all throughout the table of co-graduation. If, on the other hand, the index of co-graduation does not maintain itself at the level of unity but decreases as the age groups become more distant from one another, then this indicates that there is a systematic factor which produces the change in the excess male mortality according to years of life. It is shown then that this is the fact whether we carry out the co-graduation according to territories or according to time.

As regards territories, we find the trend of the excess male mortality with years of life specially pronounced in *Sweden*, *England* and the *Netherlands* and less so in *France*.

Regarding the trend of excess male mortality with time, an investigation by *Lexis* may also be quoted covering the mortality in *Belgium* during the year 1841–1860. The method he used for ascertaining whether the excess male mortality from year to year for a given age group remained more or less the same, or whether it differed systematically, was to calculate the standard error of a percentage in

[1] *Nora Federici*. «La Mortalità Differenziale dei due sessi e le sue possibili cause.» Statistica. Vol. X, No. 3. p. 274.

Table 9. The Male Death Rate in Age Groups, and Infant Mortality, Expressed as a Percentage of the Female Death Rate.

Period	Ages												Mortality Infant
	0–4	5–9	10–14	15–19	20–24	25–34	35–44	45–54	55–64	65–74	75–84	85+	
1841–45	117	103	94	88	104	94	101	114	111	111	109	106	122
1846–50	116	103	94	91	104	94	99	113	112	111	109	107	121
1851–55	116	102	97	90	104	97	102	118	114	112	110	106	122
1856–60	115	99	96	91	102	96	103	118	115	111	108	107	121
1861–65	115	103	98	93	106	99	109	122	118	112	109	110	122
1866–70	115	107	101	93	107	105	113	124	120	115	109	111	120
1871–75	117	107	101	96	109	108	119	128	121	114	111	110	121
1876–80	118	108	98	96	108	108	119	129	122	114	112	111	122
1881–85	118	103	96	96	101	104	117	127	122	116	113	112	122
1886–90	119	100	94	99	106	107	117	129	122	117	112	114	121
1891–95	119	99	94	101	109	108	118	128	121	115	111	110	122
1896–00	118	99	97	107	118	115	122	129	124	117	113	109	121
1901–05	119	97	96	106	120	117	121	130	128	119	115	110	122
1906–10	119	97	94	108	119	118	122	129	128	121	115	113	123
1911–15	120	101	97	109	122	126	127	131	133	124	118	115	125
1916–20	121	100	94	107	148	147	142	135	137	132	121	111	128
1921–25	124	106	98	104	114	113	129	132	133	138	120	113	130
1926–30	125	110	105	107	112	111	132	137	136	131	123	117	131
1931–35	126	110	105	110	114	107	125	141	139	132	124	114	130
1936–40	128	115	112	115	120	115	129	147	152	136	130	113	129
1941–45	126	129	124	128	207	169	147	153	165	144	130	109	127
Average	116	105	99	102	117	112	120	129	127	121	115	111	124
1946	130	129	128	130	149	113	127	154	168	144	127	113	129
1947	131	139	128	134	113	109	126	157	175	149	130	121	127
1948	128	142	116	116	198	106	129	155	177	153	129	115	126
1949	130	140	129	132	111	106	126	156	177	150	126	115	128

two ways: first by the combinatorial formula $\sqrt{\frac{\overline{Pq}}{n}}$ and then by the root-mean-square formula. A difference between the two results, provided it proves significant, is then taken as an indication that the phenomenon has changed with time. The following table shows the result.

Table 10. Male Mortality per 1000 Females.

Age	Mean	Range	R	r	Q
Stillborn	1348	(1281–1410)	23,4	23,6	0,99
0– 1 M.	1359	(1316–1417)	18,5	22,1	0,84
1– 2 M.	1323	(1237–1445)	42,4	37,1	1,15
2– 3 M.	1253	(1158–1390)	36,2	40,8	0,91
3– 4 M.	1224	(1099–1394)	49,1	42,9	1,14
4– 5 M.	1284	(1174–1429)	52,7	50,6	1,04
5– 6 M.	1257	(1117–1422)	56,2	52,9	1,06
6– 9 M.	1179	(1109–1257)	34,3	30,4	1,13
9–12 M.	1085	(1014–1182)	31,1	27,8	1,12
1– 2 Y.	1028	(966–1087)	23,9	15,6	1,53
2– 3 Y.	990	(926–1065)	23,5	22,1	1,06
3– 5 Y.	947	(879–1019)	23,7	20,1	1,16
5–10 Y.	878	(821–945)	28,7	17,4	1,66
10–15 Y.	713	(620–847)	45,5	18,4	2,5
15–20 Y.	770	(685–919)	37,9	18,3	2,1
20–25 Y.	1095	(965–1234)	40,2	23,9	1,7
25–30 Y.	905	(804–1027)	32,8	21,3	1,5
30–40 Y.	826	(766–909)	29,7	13,9	2,1
40–45 Y.	943	(812–1115)	50,3	21,6	2,3
45–50 Y.	1143	(853–1468)	88,9	25,8	3,4
50–55 Y.	1124	(837–1353)	104,4	24,2	4,3
55–60 Y.	1055	(850–1305)	93,9	21,8	4,3
60–65 Y.	962	(848–1140)	64,8	18,5	3,5
65–70 Y.	913	(789–1151)	71,7	16,6	4,3
70–75 Y.	906	(766–1150)	65,9	15,9	4,1
75–80 Y.	903	(811–1019)	36,0	16,8	2,1
80–85 Y.	866	(781–940)	24.5	19,5	1,26
85–90 Y.	800	(721–904)	33,9	26,3	1,29
over 90 Y.	693	(638–831)	28,7	38,1	0,75

R – Standard error of percentage calculated by *Root-Mean-Square* formula: $R = \sqrt{\frac{\overline{\Sigma\delta^2}}{n-1}}$

r – Standard error of percentage calculated by combinatorial formula: $r = \sqrt{\frac{\overline{Pq}}{N}}$. $Q = \frac{R}{r}$.

The first column gives the mean excess male mortality per thousand, the second column the range which we encounter in the twenty years in question, the third column gives the standard error of the percentage of male births calculated by the root-mean-square formula, the fourth column the standard error calculated by the combinatorial formula, and the last column their ratio.

The results are very interesting in two respects. We first find that up to about the first year of life the ratio Q is sensibly equal to unity which is interpreted as being due to the differences of the excess male mortality with time for these age groups being not significant, but only fluctuations around one and the same mean. From one year of life onwards the ratio increases, first slowly, then faster; it reaches a peak between 40 and 60, if not 40 and 70, which indicates that for these age groups the differences are considerable and that there is a systematic trend with time. For the higher age groups above 70 we find the ratio Q again decreasing and approaching unity.

We have, therefore, on the one hand, the result reached before, that the excess male mortality has its peak in the higher age groups, between 40 and 60 or 40 and 70, and on the other hand, that for these age groups we also find a more pronounced trend with time. The explanation of this phenomenon can clearly not be sought solely in a greater exposure to environmental risk or in genetical differences between the sexes. Occupational risk may increase either with age of life or with time insofar as certain new occupations may represent a new or additional risk: but on the whole the tendency is surely one of decreasing occupational risk. Furthermore, it is a matter of common experience that the proneness to accidents in industrial occupations rather decreases with age because, as the worker becomes more experienced, he also learns how to avoid accidents.

The genetical hypothesis does not seem to be able to explain why excess male mortality should reach its peak between 40 and 60, nor why there should be a greater variability for these age groups. One would be inclined to conclude that recessive characteristics for diseases would find expression soon after birth, which is in agreement with the high excess male mortality for infants, neonatal mortality and still births.

That recessive characteristics for diseases terminating in death should wait for the age of full maturity and after, before expressing themselves is not very plausible, and it is, therefore, the third hypothesis to which we turn in order to explain the phenomenon of the differences in excess male mortality with age of life.

This hypothesis makes the metabolic and physiological differences between the sexes responsible for the phenomenon of excess male mortality.

There are certain differences, metabolic and physiological which appear early in the life of the embryo and which give rise to endocrin-

ological differences. Once these difference are established they take charge of the further differentiation between the sexes. The initial genetic constitution would seem to determine which of the two alternative types of differentiation shall occur, and with the development of the endocrinological system, maleness or femaleness becomes finally established.

These two states are among other things distinguished by definite differences in oxidation rate. Experiment has shown that the higher metabolic rate of the male renders him less resistant to unfavourable conditions and more prone to death.[1]

To borrow an expression by *Claude Bernard* about the internal environment, we may speak of the differences in that respect between the sexes, which must be due to their respective endocrinological make-up, as creating either a male or a female internal environment. It will be generally admitted that the female is, as a rule, nearer to the norm than the male. There is greater variability in males than in females due to the fact that we may speak of one predominant function of the female between the ages of 20 and 45, which is child-bearing and childrearing, whereas there is no such predominant function and occupation in the life of the male. He is therefore more liable to vary according to his activity. Such variation will lead to greater individualisation as time proceeds and will make itself fully felt in the higher ages. One might compare this with the spreading out of the radii of a circle, the distance between which near the centre is small but increases as the length of the radii increases. The increased individualisation of the human male according to occupation and activity in general will lead him far away from the norm,—if we could speak of a norm at all,—so as to make him subject to certain influences and decrease his resistance to disease. The further his activity removes him from the natural conditions of life the greater will his liability to disease on this account become. Insofar, his occupational environment must be regarded as a potent factor.

The full development of what we might call the male internal environment will not be reached before his habits become inveterate which, as is well known, is one cause of decay. In accordance with the facts we may, therefore, think of the years between 40 and 60 as representing the peak of the development of the male internal environment. After 60, high age with its deprivations in the way of

[1] *F. A. E. Crew*, l. c.

activity, acts as an equalising or levelling force which produces the drop in excess male mortality, in accordance with this explanation.

Conclusions.

On the basis of the foregoing discussion, the conclusion appears to be justified that the three hypotheses which were advanced for the explanation of the phenomenon of excess male mortality, viz. the hypothesis of increased occupational and environmental risk in the male, the genetical hypothesis and the physiological hypothesis should not be considered as mutually exclusive. Each of these explanations appears to have its proper sphere of application according to age of life and to the particular aspect of excess male mortality. The explanation of increased occupational and environmental risk for the male applies to those age groups in which the male may be said to be exposed to higher risk of dying from violence and from occupational diseases. This hypothesis does not provide a general explanation of the phenomenon in question but applies, strictly speaking, only to certain causes of death, among which violence and occupational disease are the most important, and to certain age groups.

The genetical hypothesis, which ascribes excess male mortality to the presence of recessive sex-linked or sex-limited genes in the male hereditary make-up, appears to be suitable for the explanation of infant mortality and also for the change in overall mortality with time.

The functional hypothesis affords an explanation of the phenomenon of the age specific differences in excess male mortality at a given time and of the change the excess male mortality for a given age group undergoes with time. It considers the higher degree of specialisation and individualisation in the male which reaches its peak in the higher age groups as being responsible for the peak of excess male mortality in these ages, and, furthermore, it considers the progressive individualisation of the human race as being responsible for the increase of the age specific excess male mortality with time. In this respect, however, the two hypotheses, the occupational, and the physiological must be regarded as concurrent explanations of one and the same phenomenon.

Résumé.

L'étude des phénomènes relatifs, autant que l'examen des hypothèses diverses, qui se sont proposées comme explication de la mortalité mâle excessive, nous amènent à la conclusion que les trois hypothèses, viz., l'hypothèse du risque d'occupation différentielle

et de l'environnement, l'hypothèse héréditaire et l'hypothèse fonctionnelle, ne doivent pas être considérées comme mutuellement exclusives. Chacune de ces explications paraît avoir sa sphère d'application particulière selon l'âge des personnes et selon l'aspect particulier de la mortalité mâle excessive.

Zusammenfassung.

Das Studium des Phänomens der Übersterblichkeit von Personen männlichen Geschlechts, und der hiefür beigebrachten Erklärungen, führt zu dem Ergebnis, daß die drei Hypothesen, nämlich die eines erhöhten Risikos des Mannes in seiner Beschäftigung, die Erblichkeitshypothese und die funktionale Hypothese, nicht als sich gegenseitig ausschließend angesehen werden müssen, und daß jede von ihnen ihre eigene Wirkungssphäre besitzt, gemäß dem Lebensalter der Person, und gemäß dem besonderen Aspekt der Übersterblichkeit.

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Michael Schwartz: **Heredity in Bronchial Asthma.** Munksgaard, Copenhagen, Denmark, 1952, 288 pp.

This extensive monograph deals with bronchial asthma and allergy and the genetic background of such conditions. The first part of the book, 134 pp., gives a comprehensive review of previous experimental, clinical and genetical studies of allergy. The author concludes that at present one cannot speak of allergic diseases as a well defined group. A number of studies indicate that various allergic conditions in man, notably bronchial asthma and hay fever, develop on the basis of genetic factors. However, data concerning the mode of genetic transmission and adequate risk figures for different relatives of allergic index cases are scanty or lacking.

For his own study the author has chosen to start with individuals suffering from bronchial asthma. The data comprise three sets of *propositi*, namely 191 cases with bronchial asthma (group A), 200 control *propositi* (group B) and 50 cases with baker's asthma (group C). The following groups of relatives of these *propositi* were investigated: children, parents, parents' siblings and grandparents. The postexaminations and the examinations of the relatives were to a very great extent performed by the author personally, thus securing a uniform body of data which should be well adapted to a genetic and statistic analysis. The occurrence of

the following diseases among the relatives was noted: bronchial asthma, hay fever, vasomotor rhinitis, *Besnier*'s prurigo, eczema, urticaria, *Quincke*'s edema, migraine, gastrointestinal allergy, epilepsy, ichtyosis and psoriasis, of which, as the author points out, the three last-mentioned diseases are of questionable allergic etiology. Adequate definitions of the diagnostic criteria used for each condition are given.

The genetic analysis of the data was performed according to the principles of *Weinberg*'s *propositus* method. The author finds that the incidence of asthma, vasomotor rhinitis and *Besnier*'s prurigo was significantly higher among the relatives of groups A and C as compared with the relatives of group B. He concludes that bronchial asthma is an inherited disease and that vasomotor rhinitis and *Besnier*'s prurigo (and possibly hay fever) are genetically related to asthma. An analysis of the mode of inheritance in bronchial asthma and related diseases leads to the assumption that the transmission follows the scheme of monohybrid dominance with incomplete penetrance and variable expressivity.

As the reviewer is not competent to deal with the specific problems concerning allergy, the clinical parts of the book have been accepted at their face value. However, some of the pertinent statistical procedures used by the author appear questionable.

The author does not explain in detail how he calculated the morbid risks (chapter 14). Evidently he did not follow current methods. Take tables 47 and 48 as an example. The number of onsets of bronchial asthma per survived 5-year group are put in relation to the number of individuals observed in each group and the incidence per thousand is given. So far it is correct and apparently the author has had some kind of a morbidity table in mind. Then he adds all these incidences per thousand for all the 5-year groups and obtains, e.g. for siblings of the A-propositi, a morbid risk of 236,9 per thousand. Now you can't add the probabilities for each 5-year group like that. The probability to fall ill is, of course, greater if more 5-year periods are available than if there would be only one. This cumulative probability is, however, not the sum of the probabilities for each period. It increases according to much more complex rules. For instance the probability to get an even number by throwing a dice is $\frac{1}{2}$. If you make three casts the probability to get at least one even number is not $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=1.5$, which would be absurd, but $\frac{7}{8}$. The correct way to deal with this calculation is to turn it over and calculate the chance of *not* falling ill for each 5-year period. These probabilities can be multiplied and the results will give you the total life time probability of not getting the disease. This probability subtracted form 1, finally, gives you the morbid risk. By applying the correct method to column 1 of tables 47 and 48 I obtained a morbid risk concerning bronchial asthma for the siblings of the A-*propositi* amounting to 215.5 per thousand instead of 236.9 per thousand. The difference as such is not important but this does not justify an apparently incorrect procedure. If the 5-year risks had been considerably greater, however, the differences would easily have amounted to substantial figures.

True, the author's morbid risk differences between the relatives of the asthmatic *propositi* and the relatives of the control *propositi* appear so great that one does not seriously question them, but the handling of the data has been somewhat irritating to the reviewer.

The author has no doubt collected a fine body of primary data and these will remain a most welcome contribution to our knowledge of bronchial asthma and allied disorders.

Jan A. Böök, Uppsala

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INDEX
